3.2.3. Incompressible and Almost Incompressible Materials

■ <u>Problem:</u>

Let us consider the primal VF (2) for an homogeneous, isotropic material. The corresponding bilinear form

(21) $a(u,v) = \lambda\,(\operatorname{div} u, \operatorname{div} v)_0 + 2\mu\,(\varepsilon(u), \varepsilon(v))_0$

is, in principle, $V_0 = H^1_{0,\Gamma_u}(\Omega)$-elliptic and V_0-bounded, i.e. $\exists\, \mu_1, \mu_2 = \text{const} > 0$: $(\text{meas}_2 \Gamma_u > 0)$

(22) $\begin{cases} a(v,v) \geq \mu_1 \|v\|_1^2 & \forall v \in V_0 := \{v \in H^1(\Omega) : v|_{\Gamma_u} = 0\} \\ |a(u,v)| \leq \mu_2 \|u\|_1 \|v\|_1 & \forall u,v \in V_0 \end{cases}$

<u>but</u> $\mu_2/\mu_1 \to \infty$ for $\nu \to \frac{1}{2}$ $\left(\Leftrightarrow \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \to \infty\right)$,

i.e. the PVF (2) becomes badly conditioned for almost incompressible materials like rubber (cf. Subsection 3.2.1)!

■ <u>Possible Solutions:</u>

1. Use the DHR-Formulation (2nd HR principle): A refined analysis of the Ker B-ellipticity of the bilinear form $a(\cdot,\cdot)$ gives:

$$a(\tau,\tau) \geq \alpha_1 \left(\|\tau\|_0^2 + \|\operatorname{div}\tau\|_0^2\right) = \alpha_1 \|\tau\|_{X_0}^2 \quad \forall \tau \in \operatorname{Ker} B \qquad (0)$$

with $\alpha_1 \neq \alpha_1(\nu)$, wheras

$$\lambda_{min}(D^{-1}) = \frac{1}{\lambda_{max}(D)} = \frac{1-2\nu}{E} \xrightarrow[\nu \to 1/2]{} 0 .$$