• Mixed FE-Approximation:

The results of Theorem 2.6 (= discrete version of Brezzi's theorem) and of Theorem 2.7 (= convergence theorem) for some conforme mixed FE-Approximation

σ $$X_{0h} := \operatorname{span}\{p^{(i)} : i \in \omega_{X_{0h}}\} \subset X_0 \subset X = H(\operatorname{div}, \Omega)$$

u $$\Lambda_h := \operatorname{span}\{q^{(j)} : j \in \omega_{\Lambda_h}\} \subset \Lambda = L_2(\Omega)$$

to the DMF $(18)_0$ are valid provided that

a) the discrete LBB-condition:

X_{0h} must be suff. large $\nearrow$ $$\sup_{\tau_h \in X_{0h}} \frac{(\operatorname{div} \tau_h, v_h)_0}{\|\tau_h\|_{H(\operatorname{div})}} \geq \tilde{\beta}_1 \|v_h\|_0 \quad \forall v_h \in \Lambda_h$$

b) the $V_{0h} := \{\tau_h \in X_{0h} : (\operatorname{div} \tau_h, v_h)_0 = 0 \ \forall v_h \in \Lambda_h\}$ - ellipticity condition

X_{0h} not too large wrt. Λ_h $\nearrow$ $$a(\tau_h, \tau_h) = (D^{-1}\tau_h, \tau_h)_0 \geq \tilde{\alpha}_1 \|\tau_h\|^2_{H(\operatorname{div})} \quad \forall \tau_h \in V_{0h}$$

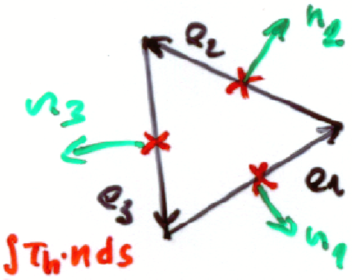
$\uparrow$

OK: $\operatorname{div} X_{0h} \subset \Lambda_h$

hold! $\Longrightarrow$ non-trivial; see [Brezzi-Fortin]!

Example: for 2D elasticity (EVZ, ESZ) or 2D Poisson-eq.

$\rightarrow$ Raviart-Thomas (1977)



$x = \int_e \tau_h \cdot n \, ds$

$\Omega \subset \mathbb{R}^2$, $\mathcal{T}_h = \{\delta_r : r \in R_h\}$ - reg. triang.

$$X_h := \{\tau_h \in (L_2(\Omega))^2 : \tau_h|_{\delta_r} = \begin{pmatrix} a_r \\ b_r \end{pmatrix} + c_r \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \ a_r, b_r, c_r \in \mathbb{R}^1, \ \tau_h \cdot n|_{\partial\delta_r} - \text{cont.}\} \subset X$$

$$\Lambda_h := \{v_h \in L_2(\Omega) : v_h|_{\delta_r} = \alpha_r \in \mathbb{R}^1\} \subset \Lambda$$