

- Then (13) and ① - ② directly give the primal mixed formulation (PMF):

$$(14) \quad \begin{cases} (D^{-1}\sigma, \tau)_0 - (\varepsilon(u), \tau)_0 = 0 & \forall \tau \in X \\ -(\sigma, \varepsilon(v))_0 = -(f, v)_0 - \int_{\Gamma_t} t^T \cdot v \, ds & \forall v \in \Delta \end{cases}$$

- We obviously have the following relationships:

$$\begin{cases} PVF (2) \Rightarrow PMF (14) \\ PMF (14) \Rightarrow PVF (2) \end{cases} \Rightarrow \begin{cases} (2) \equiv (14) \\ \exists! \Rightarrow \exists! \end{cases}$$

Theorem 3.7

- The PMF (14) can be written as abstract MVP:
→ see also Chapter 2!

Find $(\sigma, u) \in X \times \Delta$:

$$a(\sigma, \tau) + b(\tau, u) = \langle F, \tau \rangle \quad \forall \tau \in X$$

$$b(\sigma, v) = \langle G, v \rangle \quad \forall v \in \Delta$$

where

$$X = \{\sigma = \{\sigma_{ij}\} : \sigma_{ij} \in L_2(\Omega) : \sigma_{ij} = \tau_{ji}\} = L_2^{\text{sym}}(\Omega)$$

$$\Delta = \{v = \{v_i\} : v_i \in H^1(\Omega) : v = 0 \text{ on } \Gamma_u\} = H_0^1(\Omega)$$

$$a(\sigma, \tau) = (D^{-1}\sigma, \tau)_0, \quad b(\tau, u) = (\varepsilon(u), \tau)_0,$$

$$\langle F, \tau \rangle = 0, \text{ i.e. } F = 0 \in X^* \text{ given,}$$

$$\langle G, v \rangle = -\int_{\Omega} f^T v \, dx - \int_{\Gamma_t} t^T v \, ds, \text{ i.e. } G \in \Delta^* \text{ given.}$$