FEM for linear elasticity problem (2):

- The Finite-Element-Discretization of (2)

$$(2)_h \quad \text{Find } u_h \in V_{0h} \subset V_0 : \; a(u_h, v_h) = \langle F, v_h \rangle \;\; \forall v_h \in V_{0h}$$

$$\text{“(12)”} \Updownarrow \qquad \Updownarrow$$

$$\text{“}(\underline{2})_h \quad \text{Find } \underline{u}_h \in \mathbb{R}^{N_h} : \; K_h \underline{u}_h = \underline{f}_h$$

can be treated in the same way as was done in the courses NuPDE resp. NuEPDE, since the standard assumptions are fulfilled. In particular, we have

- Cea's theorem:

$$\| u - u_h \|_1 \leq \frac{\mu_2}{\mu_1} \inf_{v_h \in V_{0h}} \| u - v_h \|_1$$

- Convergence theorems: H^1, L_2, L_∞

- Spectral theorem:

$$K_h = K_h^T \text{ p.d.} : \quad \lambda_{min}(K_h) = O(h^d)$$

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$$\lambda_{max}(K_h) = O(h^{d-2})$$

$$\Longrightarrow \quad \mathrm{cond}_2(K_h) \lessapprox \mathrm{const} \cdot \frac{\mu_2}{\mu_1} h^{-2}$$

- The factor $\frac{\mu_2}{\mu_1}$ (= condition of the BVP) influences both the quality of the FE approximation u_h and the condition of the stiffness matrix K_h:

$\Longrightarrow$ Locking (FEM yields too small displacements);

$\longrightarrow$ volume locking : $\mu_2 / \mu_1 = O\left(\frac{1}{1-2\nu}\right) = O(\lambda)$

$\longrightarrow$ shear locking : $\mu_2 / \mu_1 = O(t^{-2})$

$\longrightarrow$ membran locking :