Corrolary 3.6: ($\bar{V}_0$-ellipticity)

Under the assumptions of Lemma 3.5 and under the assumptions imposed on the elastic constants in (2), the bilinear form $a(u,v) = \int_\Omega \sigma^T(u)\,\varepsilon(v)\,dx$ is V_0-elliptic, i.e.

(10) $a(v,v) \geq \mu_1 \|v\|_1^2 \quad \forall v \in \bar{V}_0 := \{v \in [H^1(\Omega)]^3 : v = 0 \text{ on } \Gamma_u\}$,

with
$$\mu_1 = \begin{Bmatrix} \lambda_{min}(D) \\ 2\mu \end{Bmatrix} c_{K2}^2 \quad \begin{matrix} \text{in general,} \\ \text{isotropic,} \end{matrix}$$

where $c_{K2}^2 = \frac{1}{2} \bar{c}_F^{-2}$ in the Dirichlet case $\bar{V}_0 = (\overset{\circ}{H}{}^1(\Omega))^3$ with the Friedrichs const.: $\|v\|_1 \leq \bar{c}_F |v|_1 \quad \forall v \in (\overset{\circ}{H}{}^1(\Omega))^3$.

Proof follows immediately from (4) and Lemmas 3.3-5 ∎

■ Now all Assumption of Lax-Milgram's theorem are fulfilled:

Theorem 3.7:

Ass.: 1. $\Omega \subset \mathbb{R}^3$ - bounded Lipschitz domain,
2. $\Gamma_u \subset \Gamma$: $meas_2(\Gamma_u) > 0$,
3. $f \in [L_2(\Omega)]^3$, $t \in [L_2(\Gamma_t)]^3$,
4. $D_{ijkl} \in L_\infty(\Omega)$: $0 < \lambda_{min}(D) \leq \lambda(D(x)) \leq \lambda_{max}(D)$ a.e.
or $\lambda, \mu \in L_\infty(\Omega)$: $0 < \underline{\lambda} \leq \lambda(x) \leq \bar{\lambda}$ a.e.,
$0 < \underline{\mu} \leq \mu(x) \leq \bar{\mu}$

St.: 1. $\exists!\, u \in \bar{V}_0 : a(u,v) = \langle F, v \rangle \quad \forall v \in \bar{V}_0$
2. The solution $u \in \bar{V}_0$ can be determined by the fixed point iteration

(11) $(u_{n+1}, v)_1 = (u_n, v)_1 + \tau(\langle F, v\rangle - a(u_n, v)) \quad \forall v \in \bar{V}_0$

and all error estimates of L&M are valid.