- For the space $V_0 := \{v \in \bar{V} = [H^1(\Omega)]^3 : v = 0 \text{ on } \Gamma_u\}$:
  $\longrightarrow$ mixed BVP

Lemma 3.5: (2nd KORN's inequality)

Ass.: 1. $\Omega \subset \mathbb{R}^3$ – bounded Lipschitz domain,
2. $\Gamma_u \subset \Gamma$: $\text{meas}_2(\Gamma_u) > 0$.

St.: $\exists$ positive constant $c_{K2} = c(\Omega, \Gamma_u) = \text{const} > 0$:

$$(8) \quad \int_\Omega \varepsilon_{ij}(v)\varepsilon_{ij}(v)\,dx \geq c_{K2}^2 \underbrace{\|v\|_{1,\Omega}^2}_{|v|_{1,\Omega}^2 \text{ (Friedrichs)}} \quad \forall v \in \bar{V}_0$$

Proof: (indirect)

- Let us assume that inequality (8) is WRONG! i.e. $\exists \{v_n\} \subset V_0$:

$$(9) \quad \|\varepsilon(v_n)\|_0^2 = \int_\Omega \varepsilon_{ij}(v_n)\varepsilon_{ij}(v_n)\,dx \leq \frac{1}{n} \text{ and } \|v_n\|_1 = 1.$$

- Since $H^1(\Omega) \hookrightarrow L_2(\Omega)$ is compact (see Nu I), there exists subsequence $\{v_{n'}\} \subset \{v_n\}$ that converges in $[L_2(\Omega)]^3$. Lemma 3.4 gives ($n' \mapsto n$)

$$\begin{aligned} c_K^2 \|v_n - v_m\|_1^2 &\leq \|\varepsilon(v_n - v_m)\|_0^2 + \|v_n - v_m\|_0^2 \leq \\ &\leq 2\|\varepsilon(v_n)\|_0^2 + 2\|\varepsilon(v_m)\|^2 + \|v_n - v_m\|_0^2 \\ &\leq \frac{2}{n} + \frac{2}{m} + \|v_n - v_m\|_0^2 \xrightarrow[n,m\to\infty]{} 0 \end{aligned}$$

- Result: $\{v_n\}$ L_2-convergent $\Rightarrow \{v_n\}$ Cauchy in H^1
  $\Rightarrow \exists u_0 \in V_0 : v_n \longrightarrow u_0$ in H^1

$$\Rightarrow \text{a)} \quad \|\varepsilon(u_0)\|_0 = \lim_{n\to\infty} \|\varepsilon(v_n)\|_0 \underset{(9)}{=} 0$$

$$\text{b)} \quad \|u_0\|_1 = \lim_{n\to\infty} \|v_n\|_1 = 1$$

$$\text{a)} \Leftrightarrow \varepsilon(u_0) = 0 \underset{\text{Lemma 3.1}}{\Longleftrightarrow} u_0 = a \times x + b \in \mathcal{R} \underset{V_0 \ni u_0|_{\Gamma_u} = 0}{\Longrightarrow} u_0 = 0 \text{ ↯ b)}$$

q.e.d.