

- For the space $V = [H^1(\Omega)]^3$:

Lemma 3.4: (Korn's inequality)

Let $\Omega \subset \mathbb{R}^d$ be a bounded Lipschitz domain.
Then there exists a constant $c_K = c_K(\Omega) = \text{const} > 0$:

$$(7) \quad \int_{\Omega} \varepsilon_{ij}(v) \varepsilon_{ij}(v) dx + \|v\|_0^2 \geq c_K^2 \|v\|_1^2 \quad \forall v \in [H^1(\Omega)]^d.$$

Proof: is very technical!

→ see literature

[] Duvaut G., Lions J.-L.:

Les équations en Mécanique
et en Physique. Dunod, Paris 1972

[] Nitsche J. A.: On Korn's second
inequality.

RAIRO Anal. Numér., 1981, v. 15, 237-248.

Remark:

Korn's inequality (7) yields the following
norm equivalence:

$$c_K \|v\|_1 \leq \|v\| := (\|\varepsilon(v)\|_0^2 + \|v\|_0^2)^{1/2} \leq \|v\|_1$$

$$\forall v \in V := [H^1(\Omega)]^3.$$