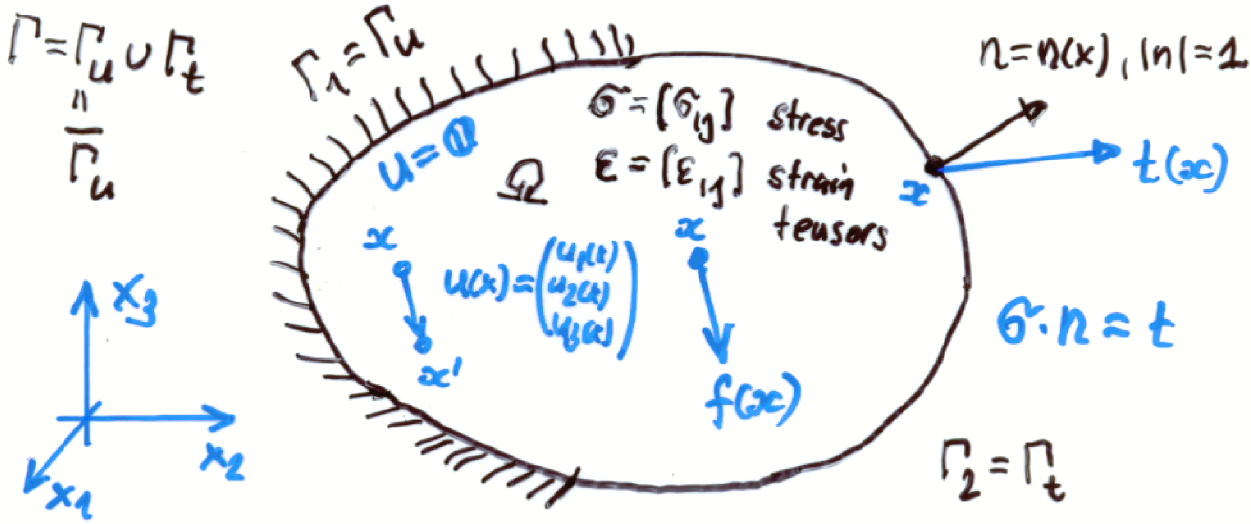
3. Linear Elasticity Problems

3.1. The Basic Equations

- The basic equations of the static linear elasticity were derived in the course
  → Mathematical Modelling in the Technique,
  see also Lecture Notes "Numerische Festkörpermechanik":



(1)

1. Equilibrium equations:

$$-\operatorname{div} \sigma = f \text{ in } \Omega \quad \Big| \quad -\sigma_{ji,j}(x) = f_i(x), \; i = \overline{1,3}, \; x \in \Omega$$

2. Geometrical relations (ε-u-relations):

$$\varepsilon = \varepsilon(u) := \tfrac{1}{2}\left(\nabla u + (\nabla u)^T\right) \quad \Big| \quad \varepsilon_{ij} = \varepsilon_{ij}(u) := \tfrac{1}{2}\left(u_{i,j} + u_{j,i}\right)$$

$i, j = 1, 2, 3$

3. Material law = Hook's law (σ-ε-relations):

$$\sigma = D\varepsilon \quad \Big| \quad \sigma_{ij} = D_{ijkl}\,\varepsilon_{kl}$$

Lamé's const.

isotropic → $D_{ijkl} = \lambda\, \delta_{ij}\delta_{kl} + \mu\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$

4. Boundary Conditions (Mixed BVP)

$\underset{\text{Dirichlet}}{u = 0}$ on Γ_u and $\underset{\text{Neumann}}{\sigma \cdot n = t}$ $(\sigma_{ji} n_j = t_i)$ on Γ_t