Exercise 2.24:

Provide the detailed algorithm (i.e. for the iterates $\underline{u}_k$ and $\underline{\lambda}_k$) of the so-called Bramble-Pasciak-CG which is nothing else but the PCG, preconditioned by $\bar{S}$, for solving (61) $S\underline{X} = \underline{F}$, and provide the corresponding iteration error estimate!

<u>Hint:</u> Defect equation: $\bar{S}\,\underline{w} = \underline{d}$ with

$$\underline{d} = F - S\underline{X}$$

$$= \begin{bmatrix} (A-A_0)A_0^{-1}\underline{f} - (A-A_0)A_0^{-1}A\underline{u} - (A-A_0)A_0^{-1}B^T\underline{\lambda} \\ BA_0^{-1}\underline{f} - \underline{g} - BA_0^{-1}(A-A_0)\underline{u} - BA_0^{-1}B^T\underline{\lambda} - C\underline{\lambda} \end{bmatrix}$$

$$= \begin{bmatrix} (A-A_0)\,A_0^{-1}(\underline{f} - A\underline{u} - B^T\underline{\lambda}) \\ B\left[A_0^{-1}(\underline{f} - A\underline{u} - B^T\underline{\lambda}) + u\right] - C\underline{\lambda} - \underline{g} \end{bmatrix}.$$

Exercise 2.25:

Write down the preconditioned Richardson-Method

$$\bar{S}\,\frac{\underline{X}_{k+1} - \underline{X}_k}{\tau} + S\,\underline{X}_k = \underline{F} := \begin{bmatrix} (A-A_0)A_0^{-1}\underline{f} \\ BA_0^{-1}\underline{f} - \underline{g} \end{bmatrix}$$

for solving (61) $S\underline{X} = \underline{F}$ in detail (for $\underline{u}_k$ and $\underline{\lambda}_k$)!
Which error estimates do you know?
Describe the relation of the Richardson-Method ($A_0 := \delta G$) and the Arrow-Hurwicz-Method (54)!