- Let

$$(70)\quad \bar{S} = \begin{bmatrix} A - A_0 & \mathbb{O} \\ \mathbb{O} & D \end{bmatrix} \text{ resp. } \bar{T} = \begin{bmatrix} I & \mathbb{O} \\ \mathbb{O} & D \end{bmatrix}$$

with the SPD Schur Complement preconditioner D satisfying the spectral equivalence inequalities

$$(71)\quad \underline{\gamma}_3 D \le BA^{-1}B^T + C \le \bar{\gamma}_3 D$$

with "good" spectral equivalence constants

$$0 < \underline{\gamma}_3 \le \bar{\gamma}_3 .$$

Then estimates (63) of Theorem 2.21 give immediately the spectral equivalence inequalities

$$(72)_T\quad \underline{\delta}\,[\bar{T}\underline{X},\underline{X}] \le [T\underline{X},\underline{X}] \le \bar{\delta}\,[\bar{T}\underline{X},\underline{X}] \quad \forall \underline{X} \in \mathbb{R}^n \times \mathbb{R}^m$$

$$\Updownarrow \qquad \qquad \text{"} \qquad \qquad \text{"} \qquad \qquad \text{"}$$

$$(72)_S\quad \underline{\delta}\,(\bar{S}\underline{X},\underline{X}) \le (S\underline{X},\underline{X}) \le \bar{\delta}\,(\bar{S}\underline{X},\underline{X}) \quad \forall \underline{X} \in \mathbb{R}^n \times \mathbb{R}^m$$

with $\quad \underline{\delta}\,\bar{S} \le S \le \bar{\delta}\,\bar{S}$

$$(73)\quad \underline{\delta} = \min\{1, \underline{\gamma}_3\}\,\underline{\gamma} \text{ and } \bar{\delta} = \max\{1, \bar{\gamma}_3\}\,\bar{\gamma}$$

i.e. $\bar{S}$ is a good preconditioner for S

if

A_0 is a good (scaled) preconditioner for A

and

D is a good (scaled) preconditioner for $BA^{-1}B^T + C$