- Then the following theorem by J. Bramble and J. Pasciak (1988) is valid:

<u>Theorem 2.21:</u> (Bramble and Pasciak, 1988)

<u>Ass.:</u> Let the assumptions
(47) Ⓐ A SPD Ⓑ B full rank Ⓒ $C = C^T \geq 0$
and the spectral equivalence inequalities
(58) $\underline{\gamma}_0 A_0 \leq A \leq \overline{\gamma}_0 A_0$ with $\underline{\gamma}_0 > 1$, A_0 SPD
be fulfilled.

<u>St.:</u> Then the following statements are valid:

1. $T = T^*$ p.d. wrt. $[\cdot,\cdot]$, i.e
$[T\underline{X}, \underline{Y}] = [\underline{X}, T\underline{Y}] \;\; \forall\, \underline{X} = \binom{u}{\lambda}, \underline{Y} = \binom{v}{\mu} \in \mathbb{R}^n \times \mathbb{R}^m$,
$[T\underline{X}, \underline{X}] > 0 \;\; \forall\, \underline{X} \in \mathbb{R}^n \times \mathbb{R}^m : \underline{X} \neq \mathbb{0}$.
2. S is SPD
3. Spectral equivalence inequalities:

$$(63)_T \quad \underline{\gamma}[\tilde{T}\underline{X}, \underline{X}] \leq [T\underline{X}, \underline{X}] \leq \overline{\gamma}[\tilde{T}\underline{X}, \underline{X}] \;\; \forall\, \underline{X} \in \mathbb{R}^n \times \mathbb{R}^m$$

$$\Updownarrow$$

$$(63)_S \quad \underline{\gamma}(\tilde{S}\underline{X}, \underline{X}) \leq (S\underline{X}, \underline{X}) \leq \overline{\gamma}(\tilde{S}\underline{X}, \underline{X}) \;\; \forall\, \underline{X} \in \mathbb{R}^n \times \mathbb{R}^m$$

$$\underline{\gamma}\,\tilde{S} \leq S \leq \overline{\gamma}\,\tilde{S}$$

with the preconditioners (regularizators)

$$\tilde{T} = \begin{bmatrix} I & \mathbb{0} \\ \mathbb{0} & BA^{-1}B^T + C \end{bmatrix} \text{ and } \tilde{S} = \begin{bmatrix} A - A_0 & \mathbb{0} \\ \mathbb{0} & BA^{-1}B^T + C \end{bmatrix}$$

and the spectral equivalence constants

$$\underline{\gamma} = \left(1 + \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} + \alpha}\right)^{-1}, \quad \overline{\gamma} = \frac{1 + \sqrt{\alpha}}{1 - \alpha}, \quad \alpha = 1 - \frac{1}{\overline{\gamma}_0}$$