Exercise 2.18:

We have shown that the preconditioned Uzawa-Algorithm is nothing else but the preconditioned Richardson method applied to $(46)_S$ $(BA^{-1}B^T + C)\lambda = BA^{-1}f - g$.
Instead of the Richardson method we can of course also use the Tschebyschev method, the gradient method, or the conjugate gradient method.
Write down the preconditioned

- Uzawa-Tschebyschev-Algorithm,
- Uzawa-Gradient-Algorithm, and
- Uzawa-CG-Algorithm,

and, in analogy to (52), the corresponding iteration error estimates!

Exercise 2.20:

Show that the preconditioned Uzawa-Alg. (51) is equivalent to the classical Uzawa-Alg. (48) applied to the preconditioned (transformed) System

$$\begin{cases} A\underline{u} + B^T D^{-0.5} \underline{\mu} = \underline{f} \\ D^{-0.5} B\underline{u} - D^{-0.5} C D^{-0.5} \underline{\mu} = D^{-0.5} \underline{g} \end{cases}$$

- Disadvantage of the Uzawa-technique:

$\Rightarrow$ System $\boxed{A\underline{u}_{k+1} = \underline{f} - B^T\underline{\lambda}_k}$ must be solved, at least, up to the discretization error, in every iteration step (Matrix $\times$ vector in (50)!)