2.3. Mixed Variational Problems with Perturbation Term

- Let

(33) $\begin{cases} \bullet\ \Lambda_c \subset \Lambda \text{ be a dense subspace, i.e. } \overline{\Lambda_c}^{\|\cdot\|_\Lambda} = \Lambda, \\ \bullet\ c(\cdot,\cdot) : \Lambda_c \times \Lambda_c \to \mathbb{R}^1 \text{ - non-negative, symmetric bilinear form; i.e.} \end{cases}$

$c(\mu,\mu) \geq 0,\ c(\mu,\nu) = c(\nu,\mu)\ \forall \mu,\nu \in \Lambda_c$

(mmc) $\Rightarrow |c(\mu,\nu)| \leq (c(\mu,\mu))^{0.5} (c(\nu,\nu))^{0.5}$

We now consider the following MVP(t) with perturbation term:

(34)
$$\boxed{\begin{array}{l} \text{Find } (u,\lambda) \in X \times \Lambda_c \\ a(u,v) + b(v,\lambda) = \langle f,v \rangle \ \forall v \in X \\ b(u,\mu) - t^2 c(\lambda,\mu) = \langle g,\mu \rangle \ \forall \mu \in \Lambda_c \end{array}}$$

under the Assumptions (3) and (33), where $t \geq 0$ is some "small" real parameter.

- The following formulations are obviously equivalent to MVP(t) (34):

1. Variational Formulation in the product space $X \times \Lambda_c$:

$$\boxed{\begin{array}{l} \text{Find } (u,\lambda) \in X \times \Lambda_c: \\ A(u,\lambda; v,\mu) = \langle f,g; v,\mu \rangle \ \forall (v,\mu) \in X \times \Lambda_c \end{array}}$$

where $A(u,\lambda;v,\mu) := a(u,v) + b(v,\lambda) + b(u,\mu) - t^2 c(\lambda,\mu)$,
$\langle f,g;v,\mu \rangle := \langle f,v \rangle + \langle g,\mu \rangle$