- The $\exists!$ of the solution $u \in V_g$: CMP (28) $\equiv$ CVP (30) does not necessarily imply the $\exists!$ of the Lagrange parameter $\lambda \in \Lambda$: $(u, \lambda) \in X \times \Lambda$ uniquely solves the MVP (2) $\equiv$ SPP (32) !

$\Rightarrow$ counter-example:

a) Consider CMP in $X = \mathbb{R}^2$: Find $\binom{x}{y} \in \mathbb{R}^2$: $(x^2 + y^2) \to \min$
s.t. $x + y = 2$

$\Rightarrow$ $\exists!$ solution $x = y = 1$

Lagrange functional: $X = \mathbb{R}^2$, $\Lambda = \mathbb{R}^1$

$$L\left(\binom{x}{y}, \lambda\right) := x^2 + y^2 + \lambda(x + y - 2)$$

$\Rightarrow$ $\exists!$ SP $x = y = 1$ and $\lambda = -2$ (mms)

b) Consider CMP in $X = \mathbb{R}^2$: Find $\binom{x}{y} \in \mathbb{R}^2$: $(x^2 + y^2) \to \min$
s.t. $x + y = 2$
$3x + 3y = 6$

$\Rightarrow$ $\exists!$ $x = y = 1$

Lagrange functional: $X = \mathbb{R}^2$, $\Lambda = \mathbb{R}^2$

$$L\left(\binom{x}{y}, \binom{\lambda}{\mu}\right) := x^2 + y^2 + \lambda(x + y - 2) + \mu(3x + 3y - 6)$$

$\Rightarrow$ SP : $x = y = 1$ and all $\binom{\lambda}{\mu} \in \mathbb{R}^2$: $\lambda + 3\mu = 2$ (mms),
i.e. Lagrange parameters are not unique !

- If the LBB-condition is additionally fulfilled, i.e.
  - standard assumptions (3) and
  - additional assumptions (27),

then all formulations are equivalent:

$$\begin{array}{ccc} (MVP) \equiv (SPP) & \overset{OK}{\Longrightarrow} \Longleftrightarrow & (CVP) \equiv (CMP) \\ \Downarrow & \Longleftarrow & \Downarrow \\ \exists!\,(u, \lambda) \in X \times \Lambda & \exists!\,\lambda \in \Lambda & \exists!\,u \in V_g \end{array}$$