- We consider now the **Lagrange functional**

$$(31)\quad \begin{cases} L(\cdot,\cdot): X \times \Lambda \longrightarrow \mathbb{R}^1 \\ L(u,\lambda) := J(u) + [\, b(u,\lambda) - \langle g,\lambda\rangle \,] \end{cases}$$

and the corresponding **Saddle Point Problem** (SPP)

$$(32)\quad \boxed{\begin{array}{l} \text{Find } (u,\lambda) \in X \times \Lambda: \\ L(u,\mu) \le L(u,\lambda) \le L(v,\lambda) \quad \forall v \in X \ \ \forall \mu \in \Lambda \end{array}}$$

lhs rhs

- **Theorem 2.12:**

$$\boxed{\begin{array}{ll} \text{Ass.:} & \text{Let the conditions (26) and (27) be fulfilled.} \\ \underline{\text{St.:}} & 1.\ \text{MVP (2)} \equiv \text{SPP (32)} \\ & 2.\ (u,\lambda) \in X \times \Lambda: \underset{\text{MVP}}{(2)} \equiv \underset{\text{SPP}}{(32)} \Rightarrow u \in X: \underset{\text{CVP}}{(30)} \equiv \underset{\text{CMP}}{(28)} \end{array}}$$

<u>Proof:</u> of St. 1. (2. is trivial)

a) SPP $\Rightarrow$ MVP

- Let $(u,\lambda) \in X \times \Lambda$: SPP (32), i.e.

rhs: $\frac{1}{2}a(u,u) - \langle f,u\rangle + [b(u,\lambda) - \langle g,\lambda\rangle] \le \frac{1}{2}a(v,v) - \langle f,v\rangle + [b(v,\lambda) - \langle g,\lambda\rangle] \quad \forall v \in X$

lhs: $\underline{\frac{1}{2}a(u,u) - \langle f,u\rangle} + [b(u,\mu) - \langle g,\mu\rangle] \le \underline{\frac{1}{2}a(u,u) - \langle f,u\rangle} + [b(u,\lambda) - \langle g,\lambda\rangle] \quad \forall \mu \in \Lambda$

- ℓ $\Leftrightarrow \quad b(u,\mu) - \langle g,\mu\rangle \le b(u,\lambda) - \langle g,\lambda\rangle \quad \forall \mu \in \Lambda$

$\Leftrightarrow \quad 0 \le b(u,\lambda-\mu) - \langle g,\lambda-\mu\rangle \quad \forall \mu \in \Lambda$

$\Leftrightarrow \quad 0 \le b(u,\nu) - \langle g,\nu\rangle \quad \forall \nu = \lambda - \mu \in \Lambda$

$\nu \mapsto -\nu \qquad =$

$\Leftrightarrow \quad b(u,\mu) = \langle g,\mu\rangle \quad \forall \mu \in \Lambda \quad \Leftrightarrow \quad u \in V_g$