

■ The CMP (28) is obviously equivalent to the following Constraint Variational Problem (GVP):

$$(30) \quad \text{Find } u \in \bar{V}_g : a(u, v) = \langle f, v \rangle \quad \forall v \in \bar{V}_0$$

This immediately follows from Theorem I.2.10  
= Theorem 1.9 (NuPDE), since

- $a(\cdot, \cdot)$  is symmetric on  $\bar{V}_0$  (even on  $X$ ),
- $a(\cdot, \cdot)$  is positive (even elliptic) on  $\bar{V}_0 \subset X$ ,
- $\bar{V}_0 \subset X$  - subspace,  $\bar{V}_g = w_g + \bar{V}_0$  - hyperplane.

■ Exercise 3.11:

Under the Assumption (26), the GVP (30)  
has a unique solution  $u \in \bar{V}_g$  ( $\exists!$ ) and  
the following a priori estimate is valid:

$$\|u\|_X \leq \frac{1}{\alpha_1} \|f\|_{X^*} + \left(1 + \frac{\alpha_2}{\alpha_1}\right) \|w\|_X \quad \forall w \in \bar{V}_g.$$

Solution: Homogenization + Lax & Milgram