- Therefore, we have
  1) $\exists !\, v_h$
  2) The apriori estimate (19) of Theorem 2.6 $(a(\cdot,\cdot) \mapsto (\cdot,\cdot) = (\cdot,\cdot)_X)$ gives

$$(*) \quad \| v_h - z_h \|_X \le \overset{=1}{\frac{\alpha_1}{\tilde{\alpha}_1}} \| u - z_h \|_X + \frac{1}{\tilde{\beta}_1} \Big(1 + \overset{=1}{\frac{\alpha_2}{\tilde{\alpha}_1}} \Big) \beta_2 \| u - z_h \|_X \quad \forall z_h \in X_h$$

with $\alpha_1 = \tilde{\alpha}_1 = 1$ and $\alpha_2 = 1$.

- For the solution $v_h \in V_{gh}$ of $(24) \equiv (23)$, we obtain by triangle inequality

$$\inf_{\tilde{v}_h \in V_{gh}} \| u - \tilde{v}_h \|_X \le \| u - v_h \|_X \le \| u - z_h \|_X + \| z_h - v_h \|_X \overset{(*)}{\le} \Big(1 + 1 + 2 \frac{\beta_2}{\tilde{\beta}_1} \Big) \| u - z_h \|_X \quad \forall z_h \in X_h$$

- <u>Result:</u>

$$\inf_{\tilde{v}_h \in V_{gh}} \| u - \tilde{v}_h \|_X \le 2 \Big(1 + \frac{\beta_2}{\tilde{\beta}_1} \Big) \inf_{z_h \in X_h} \| u - z_h \|_X$$

q.e.d.