! $v^* = Pu \in \hat{U}$

Assume ⊕ $(v^* - u, v - v^*) \geq 0 \quad \forall v \in \hat{U}$ $\quad v = \tilde{v}$

$(\tilde{v} - u, v - \tilde{v}) \geq 0 \quad \forall v \in \hat{U}$ $\quad v = v^*$

$$(v^* - \tilde{v}, \tilde{v} - v^*) = -\|\tilde{v} - v^*\|^2 \geq 0$$

i.e. $v^* = \tilde{v}$ OK

2. P is non-expansive !

$(Pu - u, v - Pu) \geq 0 \quad \forall v \in \hat{U}$ $\quad v = P\tilde{u}$

$(P\tilde{u} - \tilde{u}, v - P\tilde{u}) \geq 0 \quad \forall v \in \hat{U}$ $\quad v = Pu$

$(Pu - u, P\tilde{u} - Pu) \geq 0$

⊕ $-(P\tilde{u} - \tilde{u}, P\tilde{u} - Pu) \geq 0$

$$(Pu - P\tilde{u} - (u - \tilde{u}), P\tilde{u} - Pu) \geq 0$$

$$(Pu - P\tilde{u}, Pu - P\tilde{u}) \leq (u - \tilde{u}, Pu - P\tilde{u})$$

$$\underset{\text{Cauchy}}{\leq} \|u - \tilde{u}\| \, \|Pu - P\tilde{u}\|$$

$$\Rightarrow \boxed{\|Pu - P\tilde{u}\| \leq \|u - \tilde{u}\|}$$

q.e.d.