1.2. Nonlinear Variational Problems ($\to$ CM II)

1.2.1. Nonlinear Variational Problems with monotone and Lipschitz-continuous Operators

■ <u>Nonlinear Variational Problem in an H-space:</u>

nonlinear

(15) Ges. $u \in V_0$: $\quad a(u, v) = \langle f, v \rangle \quad \forall v \in V_0$

Linear

$$A(u) = f \quad \text{in } V_0^*$$

with $A : V_0 \to V_0^*$:

(16) $\langle A(u), v \rangle := a(u, v) \quad \forall u, v \in V_0$

■ <u>The main result:</u> ($\hat{=}$ Lax-Milgram)

<u>Ass.:</u> 0. $V_0 < V$ - subspace of H-space $V, \|\cdot\|, (\cdot, \cdot)$

1. $f \in V_0^*$
2. $A : V_0 \mapsto V_0^*$:

2a) strongly monotone, i.e. $\exists \mu_1 = \text{const} > 0$:

$$\langle A(u) - A(v), u - v \rangle = a(u, u-v) - a(v, u-v) \geq \mu_1 \|u - v\|^2 \quad \forall u, v \in V_0$$

2b) Lipschitz-continuous, i.e. $\exists \mu_2 = \text{const} > 0$:

$$\|A(u) - A(v)\|_{V_0^*} := \sup_{w \in V_0} \frac{\langle A(u) - A(v), w \rangle}{\|w\|} =$$

$$= \sup_{w \in V_0} \frac{a(u, w) - a(v, w)}{\|w\|} \leq \mu_2 \|u - v\| \quad \forall u, v \in V_0$$

<u>St.:</u> $\exists! u \in V_0$: (15) and the fixed point iteration

(17) $\quad u_{n+1} = u_n - \varrho J^{-1}(A(u_n) - f) \xrightarrow[n \to \infty]{} u \quad \text{in } V_0.$