- **Corollary 1.6:**

> If we assume only
> 1) continuity, i.e. (8) and
> 2) inf-sup-condition, i.e. (9),
>
> then
>
> (11) $A : U \mapsto (\operatorname{Ker} A^*)^\circ := \{v \in \bar{V} : a(u,v) = 0 \ \forall u \in \bar{U}\}^\circ$
>
> is an isomorphism and (9) = inf-sup is equivalent to
>
> (12) $\|Au\|_{V^*} \geq \mu_1 \|u\|_U \quad \forall u \in \bar{U}$

Proof: follows immediately from the proof of Theorem 1.5. ■

- **Corollary 1.7:** (= Lax-Milgram)

> Ass.: 0) $U = V$ - Hilbert space, $\|\cdot\|$, $(\cdot,\cdot)$
> 1) $|a(u,v)| \leq \mu_2 \|u\| \|v\| \ \forall u,v \in \bar{V}$
> 2) $a(v,v) \geq \mu_1 \|v\|^2 \ \forall v \in V$
>
> St.: $A : V \to V^*$ is an isomorphism

Proof:

1) $\Rightarrow$ (8)

2) $\Rightarrow$ (9) $\displaystyle \sup_{\substack{v \in \bar{V} \\ v \neq 0}} \frac{a(u,v)}{\|u\| \|v\|} \overset{v=u}{\geq} \frac{a(u,u)}{\|u\|^2} \geq \mu_1 > 0 \ \forall u \in \bar{U}$

2) $\Rightarrow$ (10) Choose $\underset{\neq 0}{u = v} \in U = V \Rightarrow a(u,u) \neq 0$ ■