- **The main analysis tool that is needed here = Closed Range Theorem!**

- **Theorem 1.4:** (Closed Range Theorem)

> Ass.: 1. X, Y – reflexive Banach spaces
> 2. $A \in L(X, Y)$
>
> St.: Then the following two statements are equivalent:
>
> (i) $A(X) := \operatorname{Im} A$ is closed in Y
> (ii) $A(X) = (\operatorname{Ker} A^*)^0$
>
> where
>
> $$(\operatorname{Ker} A^*)^0 := \{ y \in Y : \langle \ell, y \rangle_{Y^* \times Y} = 0 \quad \forall \ell \in \operatorname{Ker} A^* \}$$
>
> = polare of $\operatorname{Ker} A^*$

- Proof: mms
  → see [Braess] pp 117 - 119
  → see LN NuI, Satz 2.17 (← Hahn & Banach)