

■ Examples:

1. BVP for scalar elliptic 2-nd order PDE:

$$V = H^1(\Omega) = W_2^1(\Omega)$$

→ see NuPDE + NuEPDE

2. The linear elasticity problem: $V = [H^1(\Omega)]^3$

→ see Lectures on Modelling and Ch. 3!

3. Plate bending problem (KIRCHHOFF plate):

$$V = H^2(\Omega) \rightarrow \text{see Ch. 4!}$$

■ FE-Approximation: → see NuPDE + NuEPDE

(1)_h Find $u_h \in V_{0h}$: $a(u_h, v_h) = \langle f, v_h \rangle \forall v_h \in \tilde{V}_0 \subset \tilde{V}_0$



(1)_h $\underline{u}_h \in \mathbb{R}^{N_h}$: $K_h \underline{u}_h = \underline{f}_h$

■ Results: → NuPDE

1. Cea's theorem = Theorem 1.17 (NuPDE)

$$(2) \|u - u_h\|_V \leq \frac{\mu_2}{\mu_1} \inf_{v_h \in \tilde{V}_{0h}} \|u - v_h\|_V$$

discret. error

approximation error

2. Approximation theorem (NuPDE + NuEPDE)

$$(3) \inf_{v_h \in V_{0h}} \|u - v_h\|_{H^s(\Omega)} \leq \|u - \bar{I}_h u\|_{H^s(\Omega)} \leq a_{s,q} h^{q-s} \|u\|_{H^q(\Omega)}$$

$s = 0, 1$; $q = 1, 2, \dots, K$ (1D: $K=1$ )

3. Convergence theorems ($H^1, L_2, L_\infty, \dots$):

e.g. $\|u - u_h\|_{H^1(\Omega)} \leq a_{1,2} h \|u\|_{H^2(\Omega)}$ (lin. el.)

4. Direct and iterative methods for solving $K_h \underline{u}_h = \underline{f}_h$

→ NuAn + NuPDE + NuEPDE + Fast Solvers