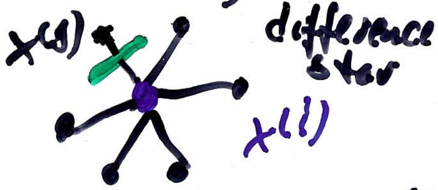
- **Result:** FV-Scheme $u^{(i)} \approx u(x^{(i)})$

$$(1)_h \quad \boxed{\begin{array}{l} -\sum\limits_{E \in \partial B(x^{(i)})}' \dfrac{u^{(j)} - u^{(i)}}{|x^{(j)} - x^{(i)}|} |E| = f^{(i)} := \int\limits_{B(x^{(i)})} f(x)\,dx \quad \forall i \in \omega_h \\ u^{(i)} = g(x^{(i)}) \quad \forall i \in \partial\omega_h = \gamma_h = \gamma_{Dh} \end{array}}$$



numerical integration

- **Convergence Analysis:**

$u : \bar{\Omega} \to \mathbb{R} : Lu = f$ in Ω, $u = g$ on $\Gamma = \partial\Omega$

$u_h : \bar{\omega}_h \to \mathbb{R} : L_h u_h = f_h$ in ω_h, $u_h = g_h$ on $\gamma_h = \partial\omega_h$

e.g. with $f_h(x^{(i)}) = f(x^{(i)}) \; \forall x^{(i)} \in \omega_h \Leftrightarrow i \in \omega_h$

$\qquad g_h(x^{(i)}) = g(x^{(i)}) \; \forall x^{(i)} \in \gamma_h \Leftrightarrow i \in \gamma_h$

$z_h = u - u_h : \bar{\omega}_h \to \mathbb{R}$ satisfies the error scheme

$L_h z_h = L_h u - L_h u_h = L_h u - f_h = L_h u - f = L_h u - Lu = \psi_h$ in ω_h approximation error

$z_h = 0$ on γ_h

$$(2) \quad \boxed{\begin{array}{l} L_h z_h = \psi_h := L_h u - Lu \text{ in } \omega_h \\ z_h = 0 \text{ on } \gamma_h \end{array}}$$

$$\|z_h\|_{X_h} = \|L_h^{-1}\psi_h\|_{X_h} \le \|L_h^{-1}\|_{[Y_h, X_h]} \|\psi_h\|_{Y_h}$$

$$\le \underbrace{\|L_h^{-1}\|_{[Y_h,X_h]}}_{\le c_s \neq c_s(h)} \underbrace{\|\psi_h\|_{Y_h}}_{\le c_a(u) h^p}$$

stability + approximation

$$\le c_s c_a(u) h^p \xrightarrow[h \to 0]{} 0 \quad \text{discr. conv.}$$

☺ **STABILITY + APPROXIMATION = DISCRETE CONV.**