- Combining (13) with $(13)_1$ and $(13)_2$ gives:

$$\mu_3 \|\tilde{u}_h - u_h\|_h^{\cancel{2}} \leq |T_1| + |T_2| \leq c\, h^{\min\{k+1,s\}-1} |u|_{H^s(T_h)} \cancel{\|\tilde{u}_h - u_h\|_h}$$

yielding

$$(14) \quad \|\tilde{u}_h - u_h\|_h \leq \frac{c}{\mu_3} h^{\min\{k+1,s\}-1} |u|_{H^s(T_h)}.$$

- (11) and (14) gives (10). q.e.d.

- **Remark 4.11:**

$$\|u - u_h\|_h \leq \sum_\delta c_\delta h_\delta^{\min\{k_\delta+1, s_\delta\}-1} |u|_{H^{s_\delta}(\delta)}$$

with $s_\delta > 3/2$, $c_\delta \leq \bar{c} = \text{const}$ $\forall \delta \in T_h$ $\forall h \in \Theta$ $(k_\delta \leq \bar{K})$

~~$\|u - u_h\|_{L_2(\Omega)} \leq c\, h^{\min\{k+1,s\}} \|u\|_{H^s(T_h)}$~~ $\downarrow$ $s \geq 2$

- **References:**

[R] B. Rivière: Discontinuous Galerkin Methods for Solving Elliptic and Parabolic Equations: Theory and Implementation. SIAM, Phil. 2008.

[PE] D. A. Di Pietro, A. Ern: Mathematical Aspects of Discontinuous Galerkin Methods. Springer-Verlag, Berlin-Heidelberg, 2012.

$\Rightarrow$ low regularity case $u \in H^s(T_h)$ with $1 < s \leq 3/2$