$$(13)_2 \quad |T_2| = \Big|\sum_e \big(\{\nabla(\overbrace{\tilde u_h - u}^{z_h})\}, [w_h]\big)_e\Big| \le$$

$$\le \sum_e \|\{\nabla z_h\}\|_e \left(\frac{h_e}{\alpha_e}\right)^{1/2} \left(\frac{\alpha_e}{h_e}\right)^{1/2} \|[w_h]\|_e$$

$$\le \Big(\sum_e \|\{\nabla z_h\}\|_e^2 \Big(\frac{h_e}{\alpha_e}\Big)\Big)^{1/2} \underbrace{\Big(\sum_e \frac{\alpha_e}{h_e}\|[w_h]\|_e^2\Big)^{1/2}}_{\lesssim \|w_h\|_h}$$

$$\le c \Big(\sum_e h_e \|\{\nabla(\tilde u_h - u)\}\|_e^2\Big)^{1/2} \|w_h\|_h$$

Let us now estimate $(\quad)$:

$$\sum_e h_e \|\{\nabla(\underbrace{\tilde u_h - u}_{=z_h})\}\|_e^2 = \sum_e h_e \Big\|\frac{1}{2}\big(\nabla z_h|_{\delta_1} + \nabla z_h|_{\delta_2}\big)\Big\|_e^2$$

$$\le \sum_e h_e \frac{1}{4}\big(\underbrace{\|\nabla z_h|_{\delta_1}\|_e}_{\overset{(9)}{\le} c\, h_{\delta_1}^{-1/2}\left(\|\nabla z_h\|_{\delta_1} + h_{\delta_1}\|\nabla^2 z_h\|_{\delta_1}\right)} + \|\nabla z_h|_{\delta_2}\|_e\big)^2$$

from Lemma 4.9.

$(a+b)^2 \le 2(a^2+b^2)$

$$\le c \sum_\delta h_\delta h_\delta^{-1}\big(\|\nabla z_h\|_\delta^2 + h_\delta^2 \|\nabla^2 z_h\|_\delta^2\big)$$

$$= c\Big[\sum_\delta \underbrace{\|\nabla(\tilde u_h - u)\|_\delta^2}_{=|\tilde u_h - u|_{1,\delta}^2 \;\lesssim\; h_\delta^{2\min\{k+1,s\}-2}|u|_{s,\delta}^2} + \sum_\delta h_\delta^2 \underbrace{\|\nabla^2(\tilde u_h - u)\|_\delta^2}_{=|\tilde u_h - u|_{2,\delta}^2 \;\lesssim\; h_\delta^{2\min\{k+1,s\}-4}|u|_{s,\delta}^2}\Big]$$

$$\le c\, h^{2\min\{k+1,s\}-2} \sum_\delta |u|_{s,\delta}^2$$

Therefore, we have

$$(13)_2 \quad \boxed{|T_2| \le c\, h^{\min\{k+1,s\}-1} |u|_{H^s(T_h)} \|w_h\|_h}$$