

Setting in (12)  $v_h = \tilde{u}_h - u_h \in \bar{V}_k(T_h)$  and using the  $\bar{V}_k(T_h)$ -ellipticity of  $a_h(\cdot, \cdot)$ , we get

$$(13) \quad \mu_3 \|\tilde{u}_h - u_h\|_h^2 \stackrel{(\#)}{\leq} a_h(\tilde{u}_h - u_h, \tilde{u}_h - u_h) =$$

$$\stackrel{(12)}{=} a_h(\tilde{u}_h - u, \underbrace{\tilde{u}_h - u_h}_{= w_h})$$

$$(3) \quad = \sum_{\delta} (\nabla(\tilde{u}_h - u), \nabla w_h)_{\delta} - \sum_e (\{\nabla(\tilde{u}_h - u)\}, [w_h])_e$$

$$- \sum_e (\underbrace{\{\nabla w_h\}}_{=0}, [\tilde{u}_h - u])_e + \sum_e \frac{\gamma_e}{h_e} (\underbrace{[\tilde{u}_h - u]}_{=0}, [w_h])_e$$

$$= T_1 + T_2 + \underbrace{T_3}_{=0} + \underbrace{T_4}_{=0} \quad \begin{matrix} \swarrow \text{Int}_{\bar{V}_k}(u) \\ \downarrow \\ \text{"0" } \leftarrow [ \tilde{u}_h - u ]_e = 0! \end{matrix}$$

• It remains to estimate  $T_1$  and  $T_2$ :

$$(13)_1 \quad |T_1| = \left| \sum_{\delta} (\nabla(\tilde{u}_h - u), \nabla w_h)_{\delta} \right|$$

$$\leq \sum_{\delta} \|\nabla(\tilde{u}_h - u)\|_{\delta} \|\nabla w_h\|_{\delta}$$

$$\leq \left( \sum_{\delta} \|\nabla(\tilde{u}_h - u)\|_{\delta}^2 \right)^{1/2} \left( \sum_{\delta} \|\nabla w_h\|_{\delta}^2 \right)^{1/2}$$

$$\leq c h^{\min\{k+1, s\} - 1} |u|_{H^s(T_h)} \cdot \|w_h\|_h$$

↑  
Approximation Theorem 3.6.