Theorem 4.10:

<u>Ass.:</u> Assume that the exact solution u of $(1)_{VF}$ belongs to $H^s(\mathcal{T}_h)$ for some $s \geq 2$ $(s > 3/2)$, and assume also that the penalty parameters α_e are large enough (cf. Lemma 4.5).

<u>St.:</u> Then there exists a positive const. $c \neq c(h)$:

$$(10) \quad \| \underset{(1)}{u} - \underset{(4)_h}{u_h^{DG}} \|_h \leq c h^{\min\{k+1, s\}-1} \|u\|_{H^s(\mathcal{T}_h)}.$$

<u>Proof:</u> Let us prove (10) for the SIPG $(\beta = -1)$.

- Let $\tilde{u}_h = \mathrm{Int}_{\tilde{V}_k(\mathcal{T}_h)}(u) = \mathrm{Int}_{V_h}(u)$: $[\tilde{u}_h]_e = 0$

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$$(11) \quad \|u - u_h\|_h \leq \underbrace{\|u - \tilde{u}_h\|_h}_{\substack{\shortparallel \\ \|u - \tilde{u}_h\|_{H^1(\Omega)} \\ \text{Theorem 3.6.} \rightarrow \wedge \\ c h^{\min\{k+1, s\}-1} |u|_{H^s(\mathcal{T}_h)}}} + \|\tilde{u}_h - u_h\|_h$$

- It remains to estimate $\|\tilde{u}_h - u_h\|_h$:

First, due to the Galerkin orthogonality, we have

$$a_h(\underset{(4)}{u} - \underset{(4)_h}{u_h}, v_h) = 0 \quad \forall v_h \in \tilde{V}_k(\mathcal{T}_h) \subset H^s(\mathcal{T}_h)$$

Consistency Th. 4.1. (a)

Therefore, we have

$$(12) \quad a_h(\tilde{u}_h - u_h, v_h) = a_h(\tilde{u}_h - u, v_h) \quad \forall v_h \in \tilde{V}_k(\mathcal{T}_h)$$