Remains to show

$$(*) = \sum_e (\{\nabla v\}, [v])_e \overset{\text{Cauchy}}{\leq}$$

$$\leq \sum_e \|\{\nabla v\}\|_e \, h_e^{1/2} \, h_e^{-1/2} \, \|[v]\|_e \overset{\text{Cauchy}}{\leq}$$

$$\leq \Big(\sum_e h_e \underbrace{\|\{\nabla v\}\|_e^2}\Big)^{1/2} \Big(\sum_e h_e^{-1} \|[v]\|_e^2\Big)^{1/2}$$

$$= \Big\|\tfrac{1}{2}\big(\nabla v|_{\delta_1} + \nabla v|_{\delta_2}\big)\Big\|_e^2$$

$$\leq \tfrac{1}{4}\big(\|\nabla v|_{\delta_1}\|_e + \|\nabla v|_{\delta_2}\|_e\big)^2$$

Lemma 4.4

$$\leq \tfrac{1}{4}\big(c h_{\delta_1}^{-1/2} \|\nabla v\|_{\delta_1} + c h_{\delta_2}^{-1/2} \|\nabla v\|_{\delta_2}\big)^2$$

$(a+b)^2 \leq 2(a^2+b^2)$

$$\leq \tfrac{1}{2}\big(c^2 h_{\delta_1}^{-1} \|\nabla v\|_{\delta_1}^2 + c^2 h_{\delta_2}^{-1} \|\nabla v\|_{\delta_2}^2\big)$$

$h_e \approx h_{\delta_2}$

$$\leq \Big(\tilde{c} \sum_\delta \|\nabla v\|_\delta^2\Big)^{1/2} \Big(\sum_e h_e^{-1} \|[v]\|_e^2\Big)^{1/2}$$

■ **Remark 4.6:**

Lemma 4.5 is also valid for $\beta = 0, +1$.