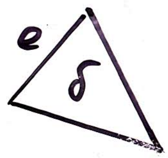
4.1.2. $V_K(\mathcal{T}_h)$ - Elliptricity and Boundedness

- **Lemma 4.4.**: $\mathcal{T}_h$ - reg. triangulation.

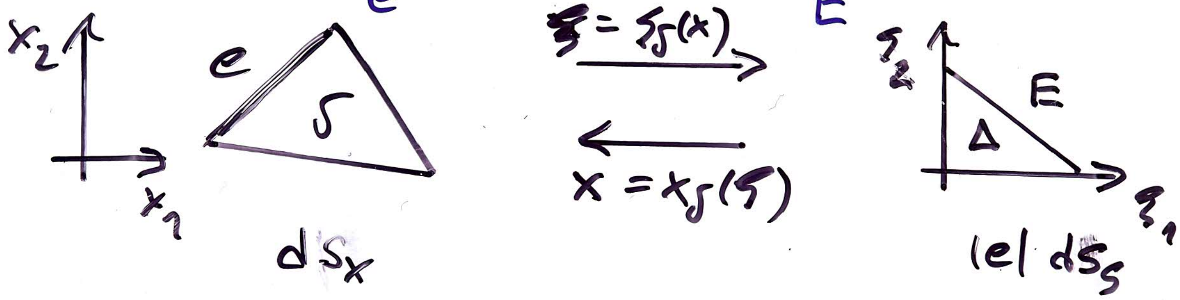
(5) $\|v\|_{L_2(e)} \le c\, h_\delta^{-1/2} \|v\|_{L_2(\delta)}$



$\forall v \in P_K(\delta)\ \forall e \in \partial\delta\ \forall \delta \in \mathcal{T}_h$

Proof: $\forall v \in P_K(\delta)$

$$\|v\|^2_{L_2(e)} = \int_e (v(x))^2 ds_x = \int_E (v(x_\delta(\xi)))^2 |e|\, ds_\xi$$



$$= |e| \int_E (\underbrace{v(x_\delta(\xi))}_{=\tilde{v}(\xi)})^2 ds_\xi = |e|\, \|\tilde{v}(\xi)\|^2_{L_2(E)}$$

$$\le |e|\, c_T^2(E)\, \|v(x_\delta(\xi))\|^2_{H^1(\Delta)}$$

trace theorem (Ch. 2): $\|\tilde{v}\|_{H^{1/2}(\partial\Delta)} \le c_T \|\tilde{v}\|_{H^1(\Delta)}$

[Riviere; p. 23: $c_T = c_T(K)$]

$$\le |e|\, c_T^2(E)\, c_{1,0}^2(\Delta)\, \|v(x_\delta(\xi))\|^2_{L_2(\Delta)}$$

Equivalence of all norms on finite-dim. spaces

$$= |e|\, c_T^2\, c_{1,0}^2 \int_\Delta (v(x_\delta(\xi)))^2 d\xi$$

$$\underset{\Delta \to \delta}{=} |e|\, c_T^2\, c_{1,0}^2 \int_\delta (v(x))^2 |J_\delta^{-1}|\, dx$$

$$\le \frac{|e|}{|\delta|} \frac{1}{2} c_T^2\, c_{1,0}^2 \|v\|^2_{L_2(\delta)} \le c^2 h_\delta^{-1} \|v\|^2_{L_2(\delta)}$$

$\frac{|e|}{|\delta|} = O(h_\delta^{-1}) \le c_1 h_\delta^{-1}$, cf. Def. 3.3. $\mathcal{T}_h$

q.e.d.