(Consistency theorem)

Theorem 4.1: Let $s > 3/2$.

(a) Assume that the weak solution u of (1), i.e. the solution of $(1)_{VF}$ $(\exists!)$, belongs to $H^s(\mathcal{T}_h)$. Then u satisfies the DG identity (4).

(b) Conversely, if $u \in H^1(\Omega) \cap H^s(\mathcal{T}_h)$ satisfies the DG identity (4), then u is also the solution of our VP $(1)_{VF}$.

Proof: (a) mms ✓ (b) mms*

DG-Scheme:

non-conform!

Let us define the DG-space $\not\subset H^1(\Omega)$

$\bar{V}_k(\mathcal{T}_h) := \{v \in L_2(\Omega) : v|_\delta \in P_k(\delta) \; \forall \delta \in \mathcal{T}_h\} \subset H^s(\mathcal{T}_h)$

Then the DG scheme reads as follows:

$$(4)_h \quad \boxed{\begin{array}{l} \text{Find } u_h = u_{DG} \in \bar{V}_k(\mathcal{T}_h): \\ a_h(u_h, v_h) = (f, v_h)_0 \quad \forall v_h \in \bar{V}_k(\mathcal{T}_h) \end{array}}$$

$$\Updownarrow$$

$$(\underline{4})_h \quad \boxed{\text{Find } \underline{u}_h \in \mathbb{R}^{N_h} : K_h \underline{u}_h = \underline{f}_h \text{ in } \mathbb{R}^{N_h}}$$

Remark 4.2:

1. The Dirichlet BC $u = 0$ on Γ is incorporated in (4) resp. $(4)_h$! (mms)
2. $\beta = -1$: SIPG = Symmetric Interior Penalty Galerkin
   $\beta = +1$: NIPG = Nonsymmetric IPG
   $\beta = 0$: IIPG = Incomplete IPG
3. $\exists! u_h : (4)_h$? L&M ?
   YES : Show $\bar{V}_k(\mathcal{T}_h)$-ell. $\wedge$ -$\mp$ of $a_h(\cdot,\cdot)$ wrt DG norm!