$$= \sum_{\delta \in \mathcal{T}_h} (\nabla u, \nabla v)_\delta - \sum_{e \in \overline{E}_h} (\{\nabla u\}, [v])_e$$

$$\sum_\delta (\cdot, vn)_{\partial\delta} = \sum_e (\cdot, [v])_e$$

$[v] = vn$ if $e \in \partial\delta \cap \Gamma$

$[v] = v_1 n_1 + v_2 n_2$ otherwise

$$= \sum_\delta (\nabla u, \nabla v)_\delta - \sum_e (\{\nabla u\}, [v])_e +$$

$[u]_e = 0$ $$+ \beta \sum_e ([u], \{\nabla v\})_e + \sum_e \frac{\alpha_e}{h_e} ([u], [v])_e$$

penalty term

where $\beta = -1, 0, 1$, and α_e are some suitably chosen param.

- Define the <u>DG-bilinear form</u>

(3) $$a_h(u,v) = a_{h,DG}(u,v) = a_{h,DG,\beta}(u,v)$$
$$:= \sum_{\delta \in \mathcal{T}_h} (\nabla u, \nabla v)_\delta - \sum_{e \in \overline{E}_h} (\{\nabla u\}, [v])_e +$$
$$+ \beta \sum_{e \in \overline{E}_h} ([u], \{\nabla v\})_e + \sum_{e \in \overline{E}_h} \frac{\alpha_e}{h_e} ([u], [v])_e$$

$$\forall u, v \in H^s(\mathcal{T}_h), \ s > 3/2$$

$\beta = -1$: $a_h(\cdot,\cdot)$ - symmetric,
$\beta = 0, 1$: $a_h(\cdot,\cdot)$ - non-symmetric,

- We can now formulate the following DG variational identity

(4) $$a_h(u,v) = (f,v)_{0,\Omega} \quad \forall v \in H^s(\mathcal{T}_h)$$

that makes sense for $s > 3/2$ and for the solution $u \in V_g \cap H^s(\mathcal{T}_h)$ with $s > 3/2$.
V_0 $(g=0)$