DG-Notations:

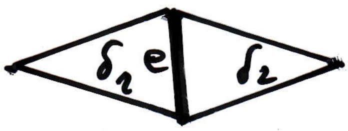
Let $\mathcal{T}_h = \{\delta_r : r \in \mathbb{R}_h\}$ be a regular triangulation of Ω.

For $s > 0$, we define the broken Sobolev spaces

$$H^s(\mathcal{T}_h) := \{ v \in L_2(\Omega) : v|_\delta \in H^s(\delta) \ \forall \delta \in \mathcal{T}_h \}, \text{ with}$$

$$\|v\|^2_{H^s(\mathcal{T}_h)} = \sum_{\delta \in \mathcal{T}_h} \|v\|^2_{H^s(\delta)}, \quad (v,u)_{H^s(\mathcal{T}_h)} := \sum_{\delta \in \mathcal{T}_h} (v,u)_{H^s(\delta)}.$$

Furthermore, we define



- the jumps (differences):

$$[\![v]\!]_e := v_1 n_1 + v_2 n_2 = \underbrace{(v_1 - v_2)}_{=:[v]_e} n_e \ \text{- vector}, \quad e \in E_h \ \text{(inner edges)}$$

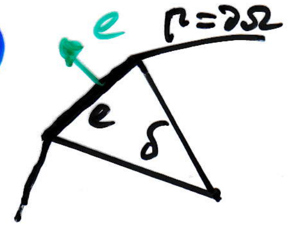
$$=: [v]_e \ \text{- scalar}$$

- the mean values (averages):

$$\{\nabla v\}_e := \tfrac{1}{2}(\nabla v_1 + \nabla v_2) \ \text{- vector}, \quad e \in E_h$$

- $e \in \partial E_h = \bar{E}_h \setminus E_h \quad (e \in \partial\delta \cap \partial\Omega)$

$$[\![v]\!] := v \cdot n, \quad \{\nabla v\}_e := \nabla v$$



DG-Formulation of (1):

- Let $u \in V_g \subset V = H^1(\Omega)$ be the solution of $(1)_{VF}$, and Let $u, v \in H^s(\mathcal{T}_h)$, $s > 3/2$, then

$$f = -\operatorname{div} \nabla u \in L_2(\delta) \quad L_2(\Omega)$$

$\nabla u \in H(\mathrm{div})$!

$$(2) \quad (f, v)_{0,\Omega} = \sum_{\delta \in \mathcal{T}_h} (f, v)_{\cancel{0},\delta} \overset{\downarrow}{=}$$

$$= \sum_{\delta} (-\operatorname{div} \nabla u, v)_\delta$$

$L_2(\partial\delta)$ since $u \in H^s(\mathcal{T}_h)$, $s > 3/2$

$$= \sum_{\delta} \left[(\nabla u, \nabla v)_\delta - (\nabla u \cdot n, v)_{\partial\delta} \right]$$

$$= \sum_{\delta \in \mathcal{T}_h} (\nabla u, \nabla v)_\delta - \sum_{\delta \in \mathcal{T}_h} (\{\nabla u\}, v \cdot n)_{\partial\delta}$$

$e \subset \partial\delta \cap \Gamma$: $(\{\nabla u\}, v \cdot n)_e = (\nabla u \cdot n, v)_e$

$e \in E_h$: $\{\nabla u\} \cdot n = \frac{1}{2}(\nabla u_1 \cdot n_1 + \nabla u_2 n_2)$

$= \nabla u_1 \cdot n_1 = \nabla u \cdot n$

$\int_e \underbrace{(\nabla u_1 n_1 + \nabla u_2 \cdot n_2)}_{=0} v \, ds = 0$

$n_1 = n_e = n$ $\quad \nabla u_1 n_1 = \nabla u_2 n_2$

$n_2 = -n_1$ $\quad$ for u: $(1)_{VF}$