- For the Galerkin solution

$(13)_h$ $\quad v_h \in X_h = \operatorname{span}\Phi_h \subset X = H^{-\frac{1}{2}}(\Gamma) : \langle w_h, V v_h \rangle = \langle w_h, f \rangle \;\forall w_h \in X_h$

$$v_h = \Phi_h \underline{v}_h = \sum_{i=1}^{n} v_i \varphi_i(x) \overset{e.g.}{=} \sum_{i=1}^{n} v_i \chi_i(x)$$

$$w_h = \varphi_k, \; k = \overline{1,n} \qquad \chi_i = \chi_{\Gamma_i}$$

$\underline{(13)}_h$ $\quad \underline{v}_h = [v_i]_{i=\overline{1,n}} \in \mathbb{R}^n : \sum_{i=1}^{n} v_i \langle \varphi_k, V v_i \rangle = \langle \varphi_k, f \rangle$
$k = \overline{1,n}$

$$K_h \underline{v}_h = \underline{f}_h$$

$$K_h = [\langle \varphi_k, V \varphi_i \rangle]_{k,i=\overline{1,n}},$$
$$\underline{f}_h = [\langle \varphi_k, f \rangle]_{k=\overline{1,n}},$$

we have:

1. CEA: $\underbrace{\|v - v_h\|_{H^{-1/2}}}_{\text{discretization error}} \leq \frac{\mu_V}{\mu_V} \underbrace{\inf_{w_h \in X_h} \|v - w_h\|_{H^{-1/2}}}_{\text{approximation error}}$

2. $K_h = K_h^T > 0$ SPD

3. K_h is a <u>dense</u> matrix !!!

4 $\varkappa(K_h) = \operatorname{cond}_2(K_h) := \frac{\lambda_{max}(K_h)}{\lambda_{min}(K_h)} = O(h^{-1}) \underset{h \to 0}{\to} \infty$