- **Properties (cf. Subsection 4.4.2):**

1. $V = V'$ (s.a.), i.e.
$$\langle w, Vv\rangle = \langle v, Vw\rangle \quad \forall v, w \in H^{-1/2}(\Gamma)$$
$$a_V(v,w) = a_V(w,v)$$
2. V is $H^{-1/2}$-elliptic (p.d.), i.e. $\exists \mu_V = \text{const} > 0$:
$$a_V(v,v) = \langle v, Vv\rangle \geq \mu_V \|v\|^2_{H^{-1/2}(\Gamma)} \quad \forall v \in H^{-1/2}(\Gamma)$$
In 2D, we suppose (15), i.e. $\operatorname{diam} \Omega < 1$.
3. $V \in L(H^{-1/2}(\Gamma), H^{1/2}(\Gamma))$, i.e. $\exists \bar{\mu}_V = \text{const} > 0$:
$$|a_V(v,v)| = |\langle v, Vv\rangle| \leq \bar{\mu}_V \|v\|_{H^{-1/2}} \|w\|_{H^{-1/2}} \quad \forall v, w \in H^{-1/2}(\Gamma)$$

- Therefore, all results of the theorem of Lax & Milgram (= Banach's fixed point theorem: $v = v + \tau J(f - Vv)$) are valid:

1. $\exists!\, v \in H^{-1/2}(\Gamma)$: $(13)_{OE}$ $Vv = f$ $\Leftrightarrow$ $(13)_{VE}$,
2. Fixed point iteration:
$$v^{(n+1)} = v^{(n)} + \tau J(f - Vv^{(n)}) \xrightarrow[n\to\infty]{} v,$$
where $J: H^{1/2} \to H^{-1/2}$ is Riesz' isomorphism.
3. Error estimates: $\tau_{opt} = \mu_V / \bar{\mu}_V^2$
$$q_{opt} = \sqrt{1-\xi}, \quad \xi = \mu_V / \bar{\mu}_V$$
$$\|v^{(n)} - v\|_{H^{-1/2}} \leq q_{opt} \|v^{(n-1)} - v\|_{H^{-1/2}}$$
$$\|v^{(n)} - v\|_{H^{-1/2}} \leq \frac{q_{opt}^n}{1 - q_{opt}} \|v^{(1)} - v^{(0)}\|_{H^{-1/2}}$$