- Using the results of Section 4.4, we can rewrite the BIE $(13)_{BIE}$ in <u>operator form</u>

$$(13)_{OE}\qquad \boxed{\text{Find } v \in X = H^{-\frac{1}{2}}(\Gamma) : \; Vv = f \text{ in } X^* = H^{1/2}(\Gamma)}$$

or as equivalent <u>variational equation</u>

$(13)_{VE}$

$$\text{Find } v \in H^{-1/2}(\Gamma) : \; \underbrace{\langle w, Vv\rangle_{H^{-\frac{1}{2}}\times H^{\frac{1}{2}}}}_{= a_V(v,w)} = \underbrace{\langle w, f\rangle}_{= \langle F, w\rangle} \;\; \forall w \in H^{-\frac{1}{2}}$$

$$\int_\Gamma w(y) \int_\Gamma E(x,y)\, v(x)\, ds_x\, ds_y = \int_\Gamma w(y) f(y)\, ds_y$$

$$=: \int_\Gamma \frac{1}{2} g_0(y) w(y)\, ds_y + \int_\Gamma w(y) \int_\Gamma \frac{\partial E}{\partial n_x}(x,y)\, g_0(x)\, ds_x\, ds_y$$

$$\sigma(y) = \frac{1}{2} \;\; \dot\forall x \in \Gamma = C^{0,1} \cap PC^k \;!!$$

where $V : H^{-1/2}(\Gamma) \longrightarrow H^{1/2}(\Gamma)$

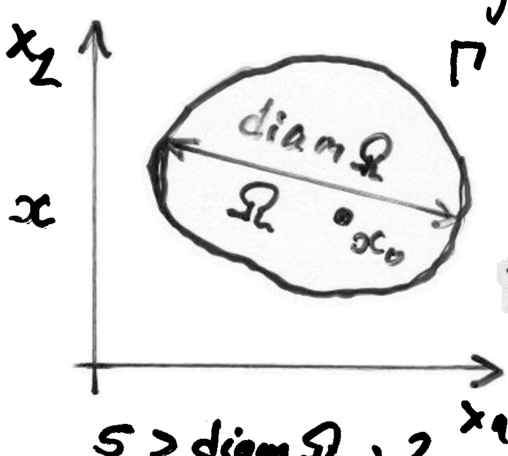
$$(14)\qquad Vv(y) := \int_\Gamma E(x,y)\, v(x)\, ds_x$$

is the single layer potential operator.

- In 2D $(d=2)$, we always suppose that

$$(15)\qquad \operatorname{diam} \Omega < 1 \quad (\Rightarrow V \text{ is p.d.})$$

which can always be obtained by scaling:



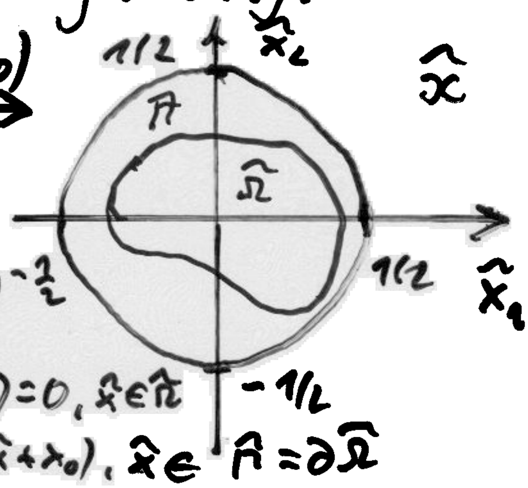
$$\hat{x} = \frac{1}{s}(x - x_0)$$

$$\frac{\partial}{\partial x_i} = \frac{\partial}{\partial \hat{x}_i}\frac{1}{s}$$

$$\hat{u}(\hat{x}) = u(\underbrace{s\hat{x} + x_0}_{= x})^{-\frac{1}{2}}$$

$$s^{-2}\Delta_{\hat{x}} \hat{u}(\hat{x}) = 0, \; \hat{x} \in \hat{\Omega}$$

$$\hat{u}(\hat{x}) = g_0(s\hat{x} + x_0), \; \hat{x} \in \hat{\Gamma} = \partial\hat{\Omega}$$



$s > \operatorname{diam} \Omega \cdot 2$