4.5. GALERKIN Discretization

- For the sake of simplicity, we consider the Dirichlet problem for the Laplace eqn.:

$$(13)_{PDE} \quad \begin{cases} -\Delta u(x) = 0, & x \in \Omega \subset \mathbb{R}^d \; (\Gamma \in C^{0,1}, d=2,3) \\ u(x) = g_D(x), & x \in \Gamma_D = \Gamma = \partial\Omega \end{cases}$$

In Subsection 4.2.4, we transformed the PDE $(13)_{PDE}$ to a weakly singular integral equation

$$(13)_{BIE} \quad \begin{cases} \text{Find } v = \frac{\partial u}{\partial n_x}\Big|_\Gamma \text{ such that} \\ \int_\Gamma E(x,y)\, v(x)\, ds_x = f(y), \quad y \in \Gamma \end{cases}$$

for determining the unknown Neumann data $v = \frac{\partial u}{\partial n_x}\big|_\Gamma$, where

$$f(y) := \sigma(y)\, g_D(y) + \int_\Gamma \frac{\partial E}{\partial n_x}(x,y)\, g_D(x)\, ds_x,$$

$$E(x,y) = \begin{cases} -\frac{1}{2\pi} \ln|x-y| & \text{for } d=2, \\ \frac{1}{4\pi} \frac{1}{|x-y|} & \text{for } d=3. \end{cases}$$