(psd)

5. D is positive semidefinite on $H^{1/2}(\Gamma)$ and positive definite (pd) on $H^{1/2}(\Gamma)|_{\ker D}$, i.e.

$$(11)_D \quad \underbrace{\langle Du, u\rangle}_{= a_D(u,u)} \geq \mu_D \|u\|^2_{H^{1/2}(\Gamma)} \quad \forall u \in H^{1/2}(\Gamma)|_{\ker D}.$$

6. For the 3D case ($d=3$), V is pd, i.e. $\exists \mu_V > 0$:

$$(11)_V \quad \langle v, Vv\rangle = a_V(v,v) \geq \mu_V \|v\|^2_{H^{-1/2}(\Gamma)} \quad \forall v \in H^{-1/2}(\Gamma).$$

7. The $H^{-1/2}$-ellipticity of V is, in general, not valid for the 2D case ($d=2$). However, if $\operatorname{diam} \Omega < 1$ (this can always be obtained by scaling of Ω !!!), the V is also p.d. (i.e. $H^{-1/2}$-elliptic) in 2D, i.e. $(11)_V$ is valid.

- <u>Mapping properties of V, K, K', D for the case of bounded Lipschitz domains $\Omega \subset \mathbb{R}^d$:</u>

For $s \in (-\frac{1}{2}, +\frac{1}{2})$, the following mappings are continuous: ($s=0$)

$$V : H^{-1/2+s}(\Gamma) \longrightarrow H^{1/2+s}(\Gamma),$$
$$K : H^{1/2+s}(\Gamma) \longrightarrow H^{1/2+s}(\Gamma),$$
$$K' : H^{-1/2+s}(\Gamma) \longrightarrow H^{-1/2+s}(\Gamma),$$
$$D : H^{1/2+s}(\Gamma) \longrightarrow H^{-1/2+s}(\Gamma).$$

The single layer potential operator V is also continuous for $s = \pm 1/2$.

In particular, $V \in L(H^{-1/2}, H^{1/2})$, i.e. $\exists \bar{\mu}_V = \text{const} > 0$:

$$(12)_V \quad \underbrace{|\langle w, Vv\rangle|}_{= a_V(v,w)} \leq \bar{\mu}_V \|w\|_{H^{-1/2}} \|v\|_{H^{-1/2}} \quad \forall w, v \in H^{-1/2}(\Gamma)$$