4.2.2. Properties of the BIO

- The operators V, K, K', D define Pseudo-Differential Operators (Ψdo) of integer orders:

$V: H^s \to H^{s+1}$ Ψdo of the order -1

$K: H^s \to H^s$ Ψdo of the order 0

$K': H^s \to H^s$ Ψdo of the order 0

$D: H^s \to H^{s-1}$ Ψdo of the order 1

provided that $\Gamma \in C^\infty$ is smooth.

- <u>Furthermore, we have (also for $\Gamma \in C^{0,1}$):</u>

1. $V = V'$ is self-adjoint, i.e.

$$(10)_V \quad \langle v, Vw\rangle_{H^{-1/2}\times H^{1/2}} = \langle w, Vv\rangle_{H^{-1/2}\times H^{1/2}} \quad \forall v,w \in H^{-1/2}(\Gamma)$$

$$\int_\Gamma\int_\Gamma E(x,y)\,w(x)\,v(y)\,ds_x\,ds_y = \int_\Gamma\int_\Gamma E(x,y)\,v(x)\,w(y)\,ds_x\,ds_y$$

$$a_V(w,v) = a_V(v,w)$$

2. $D = D'$ is self-adjoint, i.e.

$$(10)_D \quad \langle Du, v\rangle_{H^{-1/2}\times H^{1/2}} = \langle Dv, u\rangle_{H^{-1/2}\times H^{1/2}} \quad \forall u,v \in H^{1/2}(\Gamma)$$

$$-\int_\Gamma\Big(\int_\Gamma \frac{\partial^2 E}{\partial n_y \partial n_x}(x,y)\,(u(x)-u(y))\,ds_x\Big)v(y)\,ds_y =: a_D(u,v) =$$

$$= a_D(v,u) := -\int_\Gamma\Big(\int_\Gamma \frac{\partial^2 E}{\partial n_x \partial n_y}\,(v(x)-v(y))\,ds_x\Big)u(y)\,ds_y$$

3. K' is adjoint to K w.r.t. $H^0 = L_2(\Gamma)$, i.e.

$$(10)_K \quad (Ku, v)_{L_2(\Gamma)} = (u, K'v)_{L_2(\Gamma)} \quad \forall u,v \in L_2(\Gamma)$$

$$\int_\Gamma\Big(\int_\Gamma \frac{\partial E}{\partial n_y}(x,y)\,u(x)\,ds_x\Big)v(y)\,ds_y = \int_\Gamma\Big(\int_\Gamma \frac{\partial E}{\partial n_y}(x,y)\,v(y)\,ds_y\Big)u(x)\,ds_x$$

4. $D1 = -\int_\Gamma \frac{\partial^2 E}{\partial n_y \partial n_x}(x,y)\,(1-1)\,ds_x = 0$, $\ker D = \operatorname{span}\{1\}$ for $-\Delta$