- Define now the following BIO (4do)

(10) $\mathcal{V} v(y) \equiv (\mathcal{V} v)(y) := \int_\Gamma E(x,y) v(x) ds_x$ – single layer potential oper.

$K u(y) := \int_\Gamma \frac{\partial E}{\partial n_x}(x,y) u(x) ds_x$ – double layer potential operator

$K' v(y) := \int_\Gamma \frac{\partial E}{\partial n_y}(x,y) v(x) ds_x$ – adjoint double layer potential op

p.f. $\int_\Gamma$ $\quad D u(y) := -\frac{\partial}{\partial n_y} \int_\Gamma \frac{\partial E}{\partial n_x}(x,y) u(x) ds_x$ – hypersing. BIO

$$:= - \int_\Gamma \frac{\partial^2 E}{\partial n_y \partial n_x}(x,y)\,(u(x) - u(y))\, ds_x$$

(Cauchy singular!)

Trick: $\left.\begin{array}{l}\Delta u = 0 \text{ in } \Omega \\ u = 1 \text{ on } \Gamma\end{array}\right\}$ $u \equiv 1$ in $\bar{\Omega}$ & $v \equiv 0$ on Γ, $(8)_{a)}$

- Using these definitions, we can rewrite $(8)_{a)}$ and $(8)_{b)}$ in <u>operator form</u> as follows:

$(8)_{a)}$ $\quad -(\frac{1}{2} I + K) u(y) + \mathcal{V} v(y) = 0$

$(8)_{b)}$ $\quad D u(y) - (\frac{1}{2} I - K') v(y) = 0$

or in a even more compact form

$$(8) \quad \begin{bmatrix} u \\ v \end{bmatrix} = C \begin{bmatrix} u \\ v \end{bmatrix} := \begin{bmatrix} \frac{1}{2} I - K & \mathcal{V} \\ D & \frac{1}{2} I + K' \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix},$$

where C is called CALDERON projection ($C^2 = C$).

$C^2 = C$: $\Rightarrow$ interesting relations! (mms)