Remark 4.4:

1. Discretization error estimates ? Theory ?
2. The most expensive steps in the Alg. 4.3 are
   <u>Step 3:</u> matrix generation:
   $2n^2$ matrix coefficients $[a_{ij}]$, $[\tilde{b}_{ij}]$
   <u>Step 4:</u> Solution of the system $Kw=f$ with the
   - dense
   - non-symmetric

   $\left.\right\}$ matrix K of dim. $n \times n$.
3. <u>Methods for Solving the System $Kw=f$:</u>
   a) direct solvers (like Gaussian elimination):
   M = Memory $= 2n^2 + O(n)$, ops $= O(n^3)$
   b) iterative solvers (like GMRES, ... ?)
   ops $(K * w^k) = O(n^2)$
   $I(\varepsilon) =$?
   ops $= I(\varepsilon) \cdot$ ops $(K * w^k)$
   condition $(K) =$?
   preconditioners C for K ?
4. Comparison with the FEM (2D) with optimal solvers (e.g. multigrid):
   $h = O(n^{-1})$, $N_B = N_{BEM} = O(h^{-1})$, $N_P = N_{FEM} = O(h^{-2})$

	direct M	direct ops	iterative M	iterative ops	
FEM	$O(h^{-3})$	$O(h^{-4})$	$O(h^{-2})$	$O(h^{-2})$	opt.
BEM	$O(h^{-2})$	$O(h^{-3})$	$O(h^{-2})$	$\geq O(h^{-2})$	??
Data-sparse BEM	$O(h^{-1})$	$O(h^{-1})$	$O(h^{-1})$	$O(h^{-1})$	opt

$\rightsquigarrow$ with possibly additional $\log h^{-1}$!

5. Analogous: Collocation in 3D : $M = O(h^{-4})$
   ops $(K * w^k) = O(h^{-4})$

$\rightsquigarrow$ Data-sparse BEM is absolutely necessary !!!