Algorithm 4.3: BEM Collocation (2D)

1. Input of the Geometry (Γ) and the BC:

$\Gamma := \{x = x(t) \in \mathbb{R}^2 : 0 \le t < 1\}$ given,

$t^* \in [0,1)$ given s.t.:

$\Gamma_D := \{x(t) : 0 \le t \le t^*\}$, $\Gamma_N := \{x(t) : t^* < t < 1\}$

$$\left.\begin{array}{l} u(t) := u(x(t)) = g_D(x(t)),\ t \in [0, t^*] \text{ given} \\ v(t) := \frac{\partial u}{\partial n_x}(x(t)) = g_N(x(t)),\ t \in (t^*, 1) \text{ given} \end{array}\right\} \text{BC}$$

2. Discretization of the boundary Γ:

$n_D = K$ – number of nodes on the Dirichlet boundary,

$n_N = n - K$ – number of nodes on the Neumann boundary,

$n = n_D + n_N$, e.g. uniform in the parameter space

$x_1, \dots, x_{n_D}$: $x_j = x((j-1)h_D)$, $j = 1, 2, \dots, n_D$,

$x_{n_D+1}, \dots, x_n$: $x_j = x(t^* + (j - n_D - 1)h_N)$, $j = n_D + 1, \dots, n$,

$x_{n+1} = x_1$; $h_D = t^*/n_D$, $h_N = (1 - t^*)/n_N$,

$y_1, \dots, y_n$: $y_j = (x_{j+1} + x_j)/2$, $j = 1, 2, \dots, n$.

3. Generation of the system matrix K and the RHS f:

$f := 0$

for $j = 1, \dots, n_D, n_D + 1, \dots, n$ generate

$$a_j = \begin{bmatrix} a_{1j} \\ \vdots \\ a_{nj} \end{bmatrix} \text{ and } b_j = \begin{bmatrix} b_{1j} \\ \vdots \\ b_{nj} \end{bmatrix};$$

for $j = 1, \dots, n_D$ set: $K_j = a_j$, $f := f + b_j \cdot (u_D)_j$,

for $j = n_D + 1, \dots, n$ set: $K_j = -b_j$, $f := f - a_j \cdot (v_N)_j$,

4. Solve the system $K w = f$ $\qquad w = [v_D^T\ u_N^T]^T$

5. Postprocessing on the basis of the Representation formula: $\qquad y \in \Omega$

$$u_h(y) = -\sum_{j=1}^{n} u_j \int_{\Gamma_j} \frac{\partial E}{\partial n_x}(x, y)\, ds_x + \sum_{j=1}^{n} v_j \int_{\Gamma_j} E(x, y)\, ds_x$$

$\nabla_y u_h(y) = \dots$