A clever computation of the $\tilde{b}_{ii}$ by means of the so-called <u>Row-Sum-Trick</u>:

Consider the auxiliary problem

$$(6) \qquad \boxed{\begin{aligned} -\Delta u(x) &= 0\,, \quad x \in \Omega, \\ u(x) &= 1\,, \quad x \in \Gamma = \partial\Omega. \end{aligned}}$$

This auxiliary problem has obviously the unique solution $u(x) \equiv 1$ $(\forall x \in \bar{\Omega})$.
This immediately yields

$$v(x) := \frac{\partial u}{\partial n_x}(x) = 0 \quad \forall x \in \Gamma$$

(6) = special case of $(1)_{PDE}$: $\Gamma_N = \emptyset$.
The Cauchy data of (6)

$$u(x) \equiv 1 \quad \text{and} \quad v(x) \equiv 0$$

will be <u>exactly</u> approximated by piecewise constant functions, i.e.

$$u = e := (1,1,\dots,1)^T \text{ and } v = \mathbb{0}$$

must be solutions of $(5)_h$ $Bu = AV$:

$$Be = A\mathbb{0} = \mathbb{0} \qquad (B = \tfrac{1}{2}I + \tilde{B})$$

that means

$$\boxed{b_{ii} = \tfrac{1}{2} + \tilde{b}_{ii} = -\sum_{\substack{j=1\\ j\neq i}}^{n} b_{ij}\,, \quad i = 1,2,\dots,n}$$

NOW the generation of the matrices

$$B = \tfrac{1}{2}I + \tilde{B} \quad \text{and} \quad A,$$

and, therefore, of the system matrix $K = [A_D \vdots -B_N]$ and the RHS $f = B_D u_D - A_N v_N$ is completed. ∎