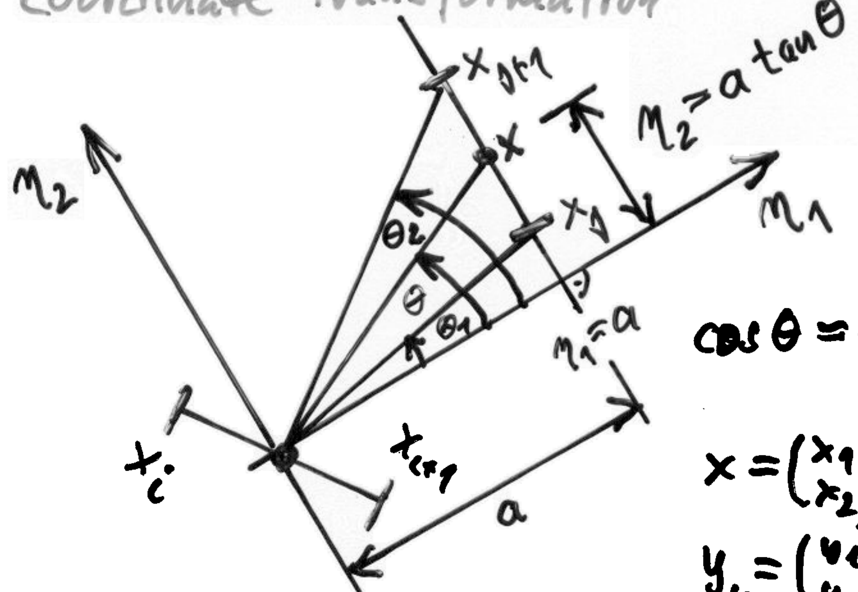
Computation of a_{ij} and $\tilde{b}_{ij}$ $(i,j = \overline{1,n})$:

1. Case: $i \neq j$ $(i,j = 1,2,\dots,n)$:

$$a_{ij} = \int_{\Gamma_j} E(x, y_i)\, ds_x = -\frac{1}{2\pi} \int_{\Gamma_j} \ln|x - y_i|\, ds_x$$

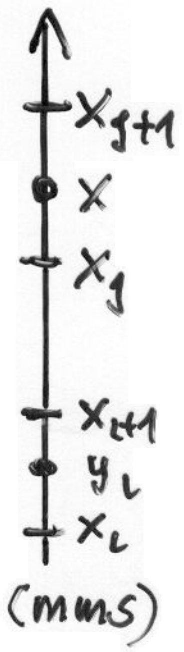
coordinate transformation



$$\cos\theta = \frac{a}{|x - y_i|}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$y_i = \begin{pmatrix} y_{1,i} \\ y_{2,i} \end{pmatrix}$$



$$|x - y_i| = a / \cos\theta$$

$$ds_x = d\eta_2 = d(a\tan\theta) = a\,\frac{d\tan\theta}{d\theta}\,d\theta = \frac{a}{\cos^2\theta}\,d\theta$$

$$a_{ij} = -\frac{1}{2\pi}\int_{\theta_1}^{\theta_2} \ln\left(\frac{a}{\cos\theta}\right)\frac{a}{\cos^2\theta}\,d\theta = -\frac{a}{2\pi}\int_{\theta_1}^{\theta_2} \ln\frac{a}{\cos\theta}\,\frac{d}{d\theta}\tan\theta\,d\theta$$

$$= -\frac{a}{2\pi}\left[\ln\left(\frac{a}{\cos\theta}\right)\tan\theta\right]_{\theta_1}^{\theta_2} + \frac{a}{2\pi}\int_{\theta_1}^{\theta_2} \frac{\cos\theta}{a}\left(\frac{-a(-\sin\theta)}{\cos^2\theta}\right)\frac{\sin\theta}{\cos\theta}\,d\theta$$

$$= -\frac{a}{2\pi}\left[\ln\left(\frac{a}{\cos\theta}\right)\tan\theta\right]_{\theta_1}^{\theta_2} + \frac{a}{2\pi}\int_{\theta_1}^{\theta_2}\tan^2\theta\,d\theta$$

$$= -\frac{a}{2\pi}\left[\tan\theta\,\ln\left(\frac{a}{\cos\theta}\right)\right]_{\theta_1}^{\theta_2} + \frac{a}{2\pi}\left[\tan\theta - \theta\right]_{\theta_1}^{\theta_2}$$

$$= -\frac{a}{2\pi}\left[\tan\theta\left(\ln\left(\frac{a}{\cos\theta}\right) - 1\right) + \theta\right]_{\theta_1}^{\theta_2} = \qquad \rho = \frac{a}{\cos\theta} = |x - y_i|$$

$$= -\frac{1}{2\pi}\left[\rho\left(\sin\theta(\ln\rho - 1) + \theta\cos\theta\right)\right]_{\rho_1 = |x_1 - x_i|,\ \theta_1}^{\rho_2 = |x_{j+1} - y_i|,\ \theta_2}$$