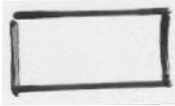


(5)_h

OR

$$(5)_h \quad \frac{1}{2} u_i + \sum_{j=1}^n u_j \tilde{b}_{ij} = \sum_{j=1}^n v_j a_{ij}, \quad i = 1, 2, \dots, n,$$

$$\text{with } \tilde{b}_{ij} = \int_{\Gamma_j} \frac{\partial E}{\partial n_x}(x, y_i) ds_x \text{ and } a_{ij} = \int_{\Omega} E(x, y_i) ds_x.$$

OR

$$(5)_h \quad \underbrace{\left(\frac{1}{2} I + \tilde{B} \right)}_{=: B} u = A v \text{ with}$$

$$u = \begin{bmatrix} u_1 \\ \vdots \\ u_k \\ u_{k+1} \\ \vdots \\ u_n \end{bmatrix}, \quad \begin{matrix} \text{Known} \\ \\ \text{Unknown} \end{matrix}, \quad v = \begin{bmatrix} v_1 \\ \vdots \\ v_k \\ v_{k+1} \\ \vdots \\ v_n \end{bmatrix}$$

OR

$$(5)_h \quad B u = A v \text{ with } u = \begin{bmatrix} u_0 \\ \vdots \\ u_n \end{bmatrix} \text{ and } v = \begin{bmatrix} v_0 \\ \vdots \\ v_n \end{bmatrix}$$

$$[B_0 : B_n] u = [A_0 : A_n] v$$

$$n \begin{bmatrix} B_0 & \dots & B_n \end{bmatrix} \begin{pmatrix} \begin{bmatrix} 0 \\ \vdots \\ u_0 \end{bmatrix} \\ + \\ \begin{bmatrix} u_0 \\ 0 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} A_0 & \dots & A_n \end{bmatrix} \begin{pmatrix} \begin{bmatrix} 0 \\ \vdots \\ v_0 \end{bmatrix} \\ + \\ \begin{bmatrix} v_0 \\ 0 \end{bmatrix} \end{pmatrix}$$

With the known Dirichlet data u_0 and v_0 , we get

$$(5)_h \quad n \underbrace{\begin{bmatrix} A_0 & \dots & -B_n \end{bmatrix}}_{=: K} \underbrace{\begin{bmatrix} v_0 \\ \vdots \\ u_n \end{bmatrix}}_{=: W} = f := \begin{bmatrix} B_0 & \dots & -A_n \end{bmatrix} \begin{bmatrix} u_0 \\ \vdots \\ v_n \end{bmatrix}$$

Result:

(5)_h

$$\text{Find } w = \begin{bmatrix} v_0 \\ \vdots \\ u_n \end{bmatrix} \in \mathbb{R}^n : K w = f$$

with the dense and non-symmetric matrix

$$K := [A_0 : -B_n]_{n \times n}$$

and the right-hand side (RHS)

$$f := B_0 u_0 - A_n v_n$$