$(5)_h$ $\boxed{}$

OR

$$(5)_h \quad \frac{1}{2} u_i + \sum_{j=1}^{n} u_j \tilde{b}_{ij} = \sum_{j=1}^{n} v_j a_{ij}, \quad i = 1, 2, \ldots, n,$$

with $\tilde{b}_{ij} = \int_{\Gamma_j} \frac{\partial E}{\partial n_x}(x, y_i)\, ds_x$ and $a_{ij} = \int_{\Gamma_j} E(x, y_i)\, ds_x$.

OR

$$(5)_h \quad \underbrace{\left(\frac{1}{2} I + \tilde{B}\right)}_{=:\,B} u = A v \quad \text{with} \quad \begin{array}{c} \Gamma_D \\ u = \\ \Gamma_N \end{array} \begin{bmatrix} u_1 \\ \vdots \\ u_K \\ u_{K+1} \\ \vdots \\ u_n \end{bmatrix} \begin{array}{l} \} \text{Known} \\ \\ \} \text{unKnown} \end{array}, \quad v = \begin{bmatrix} v_1 \\ \vdots \\ v_K \\ v_{K+1} \\ \vdots \\ v_n \end{bmatrix}$$

OR

$$(5)_h \quad B u = A v \quad \text{with} \quad u = \begin{bmatrix} u_D \\ u_N \end{bmatrix} \text{ and } v = \begin{bmatrix} v_D \\ v_N \end{bmatrix}.$$

$$[B_D \vdots B_N]\, u = [A_D \vdots A_N]\, v$$

$$n \begin{array}{c} \begin{array}{cc} K & n-K \end{array} \\ \left[B_D \vdots B_N \right] \end{array} \left(\begin{bmatrix} 0 \\ u_N \end{bmatrix} + \begin{bmatrix} u_D \\ 0 \end{bmatrix} \right) = [A_D \vdots A_N] \left(\begin{bmatrix} 0 \\ v_N \end{bmatrix} + \begin{bmatrix} v_D \\ 0 \end{bmatrix} \right)$$

With the Known Dirichlet data u_D and v_N, we get

$$(5)_h \quad n \underbrace{\begin{array}{c} \begin{array}{cc} K & n-K \end{array} \\ \left[A_D \vdots -B_N \right] \end{array}}_{=:\,K} \underbrace{\begin{bmatrix} v_D \\ u_N \end{bmatrix}}_{=:\,w} = f := [B_D \vdots -A_N] \begin{bmatrix} u_D \\ v_N \end{bmatrix}$$

Result:

$$(5)_h \quad \boxed{\text{Find } w = \begin{bmatrix} v_D \\ u_N \end{bmatrix} \in \mathbb{R}^n : \; K w = f}$$

with the dense and non-symmetric matrix

$$K := [A_D \vdots -B_N]_{n \times n}$$

and the right-hand side (RHS)

$$f := B_D u_D - A_N v_N$$