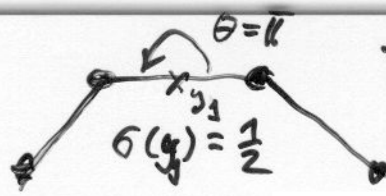

Collocation:

The integral equation

$$(5)_{BIE}\quad \frac{1}{2}u_h(y) \overset{!}{\underset{\approx}{=}} -\int_\Gamma u_h(y)\frac{\partial E}{\partial n_x}(x,y)\,ds_x + \int_\Gamma v_h(x)E(x,y)\,ds_x, \qquad y\in\Gamma$$

cannot be satisfied by chosing the unknown Cauchy data

$u_{K+1},\dots,u_n$ (Dirichlet data on Γ_N) and

$v_1,\dots,v_K$ (Neumann data on Γ_D),

since we have

$$\boxed{n \text{ unknowns for infinite many conditions } y\in\Gamma\,!}$$

Therefore, we require that the boundary integral equation $(5)_{BIE}$ should be fulfilled at n so-called collocation points

$$y_i,\quad i=1,2,\dots,n,$$

e.g. at the collocation points

$$y_i = x_i + \frac{1}{2}(x_{j+1}-x_j) = \frac{1}{2}(x_{j+1}-x_j),\quad j=1,2,\dots,n,$$

i.e. at the mid-points of the pieces Γ_j of the polygon Γ_h:

$$(5)_h\quad \boxed{\begin{gathered}\frac{1}{2}u_i = -\sum_{j=1}^{n} u_j\int_{\Gamma_j}\frac{\partial E}{\partial n_x}(x,y_i)\,ds_x + \sum_{j=1}^{n} v_j\int_{\Gamma_j}E(x,y_i)\,ds_x,\\ i=1,2,\dots,n \text{ with given } u_j\ (j=\overline{1,K}) \text{ and } v_j\ (j=\overline{K+1,n}).\end{gathered}}$$

$\Rightarrow$ n conditions for n unknowns

$$u_j\ (j=\overline{K+1,n}),$$
$$v_j\ (j=\overline{1,K}).$$