Piecewise constant (?) approximations of the Cauchy data:

For the functions

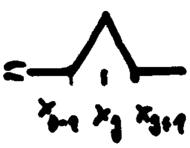
u (Dirichlet data) and

v (Neumann data),

we use piecewise constant approximations:

$$u_j \approx u(x), \; x \in \Gamma_j = [x_j, x_{j+1}) \quad \forall j = \overline{1,n}:$$

$$\chi_j(x) \overset{?}{\rightsquigarrow} \varphi_j(x) =$$



YES

$$u_h(x) = \sum_{j=1}^{n} u_j \chi_j(x) \approx u(x) \;\forall x \in \Gamma$$

$$v_j \approx v(x) := \frac{\partial u}{\partial n_x}(x), \; x \in \Gamma_j \;\forall j = \overline{1,n}$$

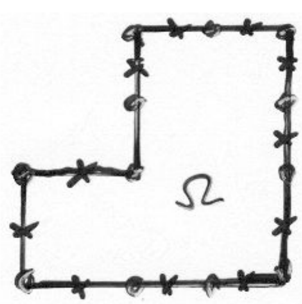
$$v_h(x) = \sum_{j=1}^{n} v_j \chi_j(x) \approx v(x) \;\forall x \in \Gamma,$$

where the values $u_1, \dots, u_K$ and $v_{K+1}, \dots, v_n$ are supposed to be Known from the boundary conditions:

e.g. $u_j = g_D(x_j)$ or $u_j = \frac{1}{2}(g_D(x_j) + g_D(x_{j+1}))$, $j = \overline{1,K}$

$v_j = g_N(x_j)$ or $v_j = \frac{1}{2}(g_N(x_j) + g_N(x_{j+1}))$, $j = \overline{K+1,n}$

or



$$u_j = g_D\left(\frac{x_j + x_{j+1}}{2}\right) = g_D(y_j), \; j = 1, \dots, K$$

$$v_j = g_N\left(\frac{x_j + x_{j+1}}{2}\right) = g_N(y_j), \; j = K+1, \dots, n$$

if $\Gamma = \Gamma_h \in PC^1 \cap C^{0,1}$ (polygonal)