- From $(5)_{PDE}$ and the representation formula (3), we immediately obtain the singular boundary integral equation $(5)_{BIE}$ for determining the unknown Cauchy data:

$$(5)_{BIE}\quad\begin{cases} \text{Find } u := u|_{\Gamma_N} \text{ and } v = t := \frac{\partial u}{\partial n_x}|_{\Gamma_D}: \\ \int_{\Gamma_N} u(x)\frac{\partial E}{\partial n_x}(x,y)\,ds_x - \int_{\Gamma_D} v(x)E(x,y)\,ds_x = f_D(y),\ y \in \Gamma_D, \\ \underbrace{\tfrac{1}{2}}_{=\sigma(y)} u(y) + \int_{\Gamma_N} u(x)\frac{\partial E}{\partial n_x}(x,y)\,ds_x - \int_{\Gamma_D} v(x)E(x,y)\,ds_x = f_N(y),\ y \in \Gamma_N \end{cases}$$

where

$$f_D(y) = -\frac{1}{2}g_D(y) - \int_{\Gamma_D} g_D(x)\frac{\partial E}{\partial n_x}(x,y)\,ds_x + \int_{\Gamma_N} g_N(x)\,E(x,y)\,ds_x,$$

$$f_N(y) = -\int_{\Gamma_D} g_D(x)\frac{\partial E}{\partial n_x}(x,y)\,ds_x + \int_{\Gamma_N} g_N(x)\,E(x,y)\,ds_x.$$