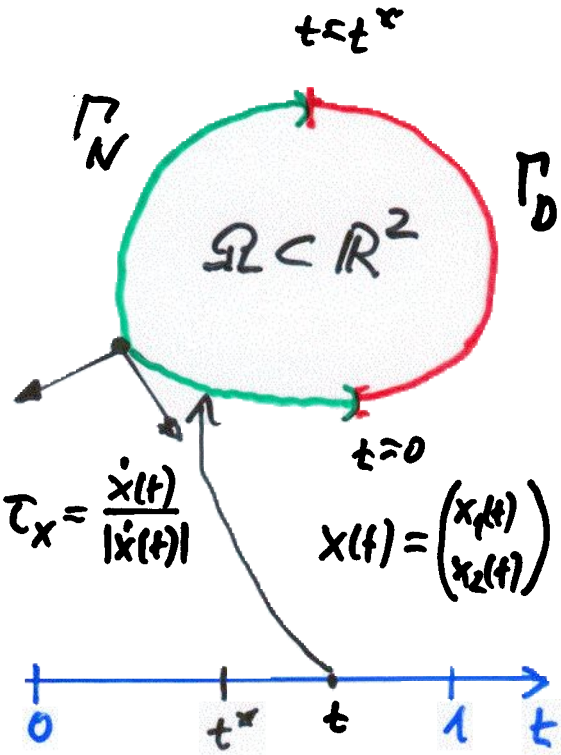
4.3. Collocation Methods

- For the sake of simplicity, we consider the 2D (analogous 3D) mixed BVP for the Laplace equation:

$$(5)_{PDE}\quad \begin{cases} -\Delta u(x) = 0, & x \in \Omega \\ u(x) = g_D(x), & x \in \Gamma_D \\ \frac{\partial u}{\partial n_x}(x) = g_N(x), & x \in \Gamma_N \end{cases}$$



$$n_x = \frac{1}{|\dot{x}(t)|}\begin{pmatrix} \dot{x}_2(t) \\ -\dot{x}_1(t) \end{pmatrix} \quad \tau_x = \frac{\dot{x}(t)}{|\dot{x}(t)|} \quad x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

<u>Ass.:</u> $|\dot{x}(t)| \geq \varkappa > 0$

$x(\cdot) \in C^1_{Per.}[0,1]$, $x(0) = x(1)$

where

$$\Gamma = \partial\Omega = \left\{ x = x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \in \mathbb{R}^2 : 0 \leq t \leq 1 \right\}$$

$$= \Gamma_D \cup \Gamma_N \in C^1_{Per}[0,1],$$

$\Gamma_D = \{ x(t) \in \mathbb{R}^2 : 0 \leq t \leq t^* \}$ – Dirichlet boundary,

$\Gamma_N = \{ x(t) \in \mathbb{R}^2 : t^* < t < 1 \}$ – Neumann boundary,

$t^* = 0$: $\Gamma_D = \emptyset \wedge \Gamma_N = \Gamma$ $\Downarrow$ Neumann BVP,

$t^* = 1$: $\Gamma_D = \Gamma \wedge \Gamma_N = \emptyset$ $\Downarrow$ Dirichlet BVP.

Generalization: $x(\cdot) \in PC^1_{per}[0,1] \cap C^{0,1}_{per}[0,1]$ OK