4. Boundary Element Methods

4.1. Introduction

■ The

BEM = Boundary Element Method

start from a Boundary Integral Formulation of a (second-order) elliptic BVP given in some computational domain $\bar{\Omega} = \Omega \cup \Gamma$. There are different methods (direct, indirect) for transforming an elliptic BVP given in $\bar{\Omega}$ to a Boundary Integral Equation on $\Gamma = \partial \Omega$.

■ In the direct approach, using

Green's formula for the considered PDE

and the

Fundamental solution of the corresponding differential operator, we obtain an identity for the boundary values, more precisely, for the so-called Cauchy-data ($u|_\Gamma$ and $\frac{\partial u}{\partial n}|_\Gamma$).

■ Together with the given boundary condition we obtain an equation on the boundary $\Gamma = \partial \Omega$
$\Rightarrow$ Boundary Integral Equation
for determining the unknown Cauchy-data.

● Knowing the Cauchy-data, we can explicitly represent the solution of the BVP in $\bar{\Omega}$ on the basis of the so-called representation formula.