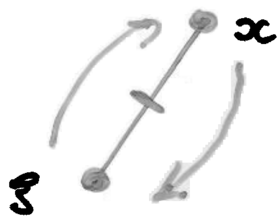
$$(27)\quad (L_h z, z) = \sum_{x\in\omega}\left(-\sum_{\xi\in S'(x)} \bar a(x_\xi)\frac{z(\xi)-z(x)}{h(x,\xi)} s(x_\xi)\, z(x)\right) + \sum_{x\in\omega} \bar c(x) z^2(x) H(x)$$

$$L_h z(x) = -\frac{1}{H(x)}\sum_{\xi\in S'(x)} \bar a(x_\xi)\frac{z(\xi)-z(x)}{h(x,\xi)} s(x_\xi) + \bar c(x) z(x)$$

$$= \sum_{x\in\omega}\sum_{\xi\in S'(x)} \bar a(x_\xi)\left(-\frac{z(\xi)-z(x)}{h(x,\xi)}\,\frac{z(x)}{h(x,\xi)}\right) h(x,\xi) s(x_\xi) + \sum_{x\in\omega} \bar c(x) z^2(x) H(x)$$



$2x \qquad h(x,\xi) = h(\xi,x)$

$$-(z(\xi)-z(x))\,z(x) - (z(x)-z(\xi))\,z(\xi)$$
$$= z^2(x) - 2z(x)z(\xi) + z^2(\xi) = (z(\xi)-z(x))^2$$

$$= \sum_{x_\xi} \bar a(x_\xi)\underbrace{\left(\frac{z(\xi)-z(x)}{h(x,\xi)}\right)}_{=:\, z_h(x_\xi)}^2 H'(x_\xi) + \sum_{x\in\omega} \bar c(x) z^2(x) H(x)$$

$$\geq \tilde\mu_1 \; \|z\|^2_{W_2^1(\omega)}$$

with $\tilde\mu_1 = \begin{cases} \min\{\bar a_1, \bar c_1\}, & \text{if } \bar c(x) \geq \bar c_1 = \text{const} > 0 \ \forall x\in\omega_h \ \forall h\in\Theta, \\ \bar a_1 (1+\tilde c_F^2)^{-1}, & \text{if } \bar c(x) \geq 0 \ \forall x\in\omega_h \ \forall h\in\Theta, \end{cases}$

where $\bar a_1 = \text{const} > 0$: $\bar a(x_\xi) \geq \bar a_1 = \text{const} > 0 \ \forall x_\xi \in \bar\omega_h \ \forall h\in\Theta$,

$\tilde c_F = \text{const} > 0$: Constant from the discrete Friedrichs ineq.:

$$\boxed{\|z\|_{L_2(\omega)} \leq \tilde c_F \, |z|_{W_2^1(\omega)} \ \forall z \in \overset{\circ}{W}{}_2^1(\omega_h) \ \forall h\in\Theta}$$

<u>Proof:</u> (mms)

Hint: Use the known Friedrichs inequality for

$\tilde z_h \in V_{0h} \subset \overset{\circ}{W}{}_2^1(\Omega)$, $\tilde z_h \longleftrightarrow z$:

$$\|\tilde z_h\|_{L_2(\Omega)} \leq c_F \, |\tilde z_h|_{W_2^1(\Omega)} \ \forall \tilde z_h \in V_{0h}.$$